Exact Synthesis of Broadband Three-Line Baluns
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Abstract—Three types of three-line baluns are studied in this paper. The equivalent circuits of these baluns are derived using the model of coupled transmission lines. The balun designs with Chebyshev response can then be exactly synthesized. A complete design procedure of the baluns with second-, third-, and fourth-order insertion loss functions is presented. Balun designs with wide bandwidths of more than 2 to 1 are achievable by this method. Some guidelines for practical realization are also suggested. A third-order balun design with a fractional bandwidth of 80\% is given to demonstrate the proposed procedure.

Index Terms—Baluns, circuit synthesis, distributed parameter filters, impedance matching.

I. INTRODUCTION

In microwave circuits such as balanced mixers, push–pull amplifiers, and antennas, baluns are needed for the transformation between the balanced and unbalanced ports. Among the various kinds of balun, Marchand balun [1] is rather popular due to its outstanding amplitude and phase balance. The synthesis method of Marchand balun with ideal Chebyshev response had been presented in [2] and [3]. Planar realizations of Marchand balun are now widely used in microwave integrated circuits and microwave monolithic integrated circuits [4]. They are generally composed of two sections of two-coupled lines. The studies on their size reduction had also been reported in several studies (see [5] and [6]).

On the other hand, a different class of planar baluns, which uses only one section of three-coupled lines, as shown in Fig. 1, promises a more compact realization. In this figure, port 1 is the unbalanced port. Port 2 and port 3 are the balanced ports. $M_1$ and $M_2$ are the additional input and output matching networks. The balun in Fig. 1(a), which is called a type-I balun in this paper, was studied in [7] and [8]. Its coaxial realization can be found as a type 4d balun in [9]. Besides the connection and terminations at the ends, this three-line balun was formed by direct combination of two identical two-coupled-line sections. Therefore, these three-coupled lines are symmetrical. However, this restriction is found unnecessary. The second and third kinds of three-line baluns are shown in Fig. 1(b) and (c), which are called type-II and type-III baluns in this paper, respectively. Coaxial baluns of these two types can be found as type 3 and type 4c baluns in [9]. A planar circuit implementation of type-III balun was studied in [10]. Nevertheless, the equivalent circuits of these two types of baluns are not fully addressed. Furthermore, the exact synthesis procedure for these three types of baluns is not yet available.

In this paper, three-line baluns are studied using the network model of generalized TEM coupled lines. It is found that the coupled lines in type-I and type-II baluns are not required to be symmetrical. Therefore, more configurations, especially those using multilayer structures, are possible. A synthesis procedure is then developed based on the derived equivalent circuit. With the additional input and output transmission-line sections, it is found that second-, third-, and fourth-order of Chebyshev responses can be realized. These baluns can be designed to have wide bandwidths of more than 2 to 1. Some design guidelines for practical realization are also presented. A design example of a third-order balun with a fractional bandwidth of 80\% is given at the end of this paper to demonstrate the proposed synthesis method.

II. EQUIVALENT CIRCUITS OF THREE-LINE BALUNS

To derive the equivalent circuit, the network model of coupled transmission lines in [11] is used. The equivalent circuit of type-I three-line balun with its corresponding terminations is shown in Fig. 2. The inductors and thick lines represent short-
Ports need to be identical and the two transformers should be the same, except for a 180° phase difference. Since the three short-circuited stubs in Fig. 4(b) can be combined through the transformers to form a single short-circuited stub connected to port 1, they will not affect the condition of balanced outputs. Therefore, the sufficient conditions for type-I balun to have balanced outputs are

$$Y_{11} = Y_{33}\quad (4)$$

$$Y_{11} - Y_{12} = Y_{13} + Y_{23}.\quad (5)$$

The above equations imply that the capacitance between line 1 (line 3) and ground should be equal to that between line 2 (line 1) and line 3 (line 2), i.e.,

$$\nu(C_{11} - C_{12} - C_{13}) = \nu C_{23}\quad (6)$$

$$\nu(C_{33} - C_{13} - C_{23}) = \nu C_{12}.\quad (7)$$

The three-coupled lines need not be symmetrical. To synthesize the balun, the equivalent circuit shown in Fig. 4(a) will be further simplified. Under the conditions of (4) and (5), the middle of the short-circuited stub with characteristic admittance of $Y_{14}$ between port 2 and port 3 can be treated as a virtual ground. Therefore, it can be separated into two equal parts and then be absorbed by the short-circuited stubs connected to port 2 and port 3. Furthermore, the short-circuited stub attached to port 1 in Fig. 4(a) is also divided into two parts, $Y_{1L}$ and $Y_{1R}$, so that the input admittances looking at port 1 to the left-hand side and to the right-hand side would be the same. With the help of equivalent circuit shown in Fig. 4(b), the admittances $Y_{1L}$ and $Y_{1R}$ can be obtained by solving (8), shown at the bottom of this page, and the results are

$$Y_{1L} = \frac{1}{2}(Y_{11} - Y_{12} - Y_{13} - Y_{23})\quad (9)$$

$$Y_{1R} = \frac{1}{2}(Y_{11} - Y_{12} + 3Y_{13} + 3Y_{23})\quad (10)$$

as shown in Fig. 4(c). Now the circuits on the two sides are identical two-port networks. Therefore, the synthesis of a three-port balun circuit is reduced to the synthesis of a two-port network, which will be discussed in Section III, as shown in Fig. 4(d). It should be noted that in this two-port network model, the load connected to port 2 should be twice the admittance that is connected to port 2.

Follow the above procedure and use the network transformation in [11], the sufficient conditions for balanced outputs are found as

$$Y_{11} - Y_{12} = Y_{33} - Y_{23}\quad (11)$$

$$Y_{22} - Y_{12} = Y_{23}\quad (12)$$

$$\begin{align*}
\left(\frac{Y_{11} - Y_{13}}{Y_{11} - Y_{12}}\right) & \left(\frac{Y_{11} - Y_{12}}{Y_{11} - Y_{13}}\right)^{2} + Y_{1L} = \left(\frac{Y_{13} - Y_{33}}{Y_{13} + Y_{23}}\right) \left(\frac{Y_{13} + Y_{23}}{Y_{33} - Y_{13}}\right)^{2} + Y_{1R} \\
\end{align*}$$

(8)
Fig. 4. (a)–(c) Equivalent circuits of type-I balun. (d) Network model for balun synthesis.

Fig. 5. Equivalent network models of: (a) type-II and (b) type-III baluns.

for the type-II balun, and

\[ Y_{11} = Y_{33}. \]  \hspace{1cm} (13)

for the type-III balun, respectively.

Equations (11) and (12) are equivalent to

\[ \nu(C_{10} + C_{13}) = \nu(C_{30} + C_{13}) \]  \hspace{1cm} (14)

\[ \nu C_{20} = 0 \]  \hspace{1cm} (15)

which means that the capacitance between line 1 and ground should be equal to that between line 3 and ground, and line 2 needs to be electrically shielded from ground, just as type 3 and type 10a baluns in [9]. The three-coupled lines are also not required to be symmetrical. For the type-III balun, the condition of (13) can be satisfied by

\[ 2Y_{12} = Y_{13} \]  \hspace{1cm} (16)

\[ 2Y_{12} = Y_{13} \]  \hspace{1cm} (17)

It then becomes a type 4c balun in [9], and its planar circuit implementation in [10]. They have line 3 electrically shielded from the other two lines by ground and line 1 electrically shielded from ground by line 2. It should be noted that there are still many other possibilities to satisfy (13). To design these two types of three-line baluns, one may follow the same method outlined above and find the final equivalent circuits as shown in Fig. 5.

They have the same network configurations as that in Fig. 4(d). Therefore, the syntheses of these three types of three-line baluns are the same. The network model in Fig. 5(b) also shows that the three-coupled lines of the type-III balun should not be symmetrical because the admittance of the unit element \( Y_{23} - Y_{12} \) will be zero. This will result in an all-stop response.

III. SYNTHESIS PROCEDURE OF THE THREE-LINE BALUNS

The synthesis procedure of the three-line baluns is similar to that of the broadband matching design for a load composed of a conductance shunted by a short-circuited stub [14]. After applying the Kuroda’s identity of the second kind, the left-hand-side short-circuited stub of the network model shown in Fig. 4(d) can be transformed to the right-hand side, and the result is correspondent with the matching network, which has a second-order insertion loss function. Furthermore, with the additional input and output transmission-line sections, the baluns with third- and fourth-order insertion loss functions can also be obtained, as shown in Fig. 6, where the corresponding characteristic admittances \( Y_A, Y_B, \) and \( Y_C \) for type-I to type-III
TABLE I

<table>
<thead>
<tr>
<th>Balun</th>
<th>( Y_a )</th>
<th>( Y_b )</th>
<th>( Y_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-I</td>
<td>( Y_{a0} - Y_{i0} )</td>
<td>( 2(Y_{b0} - Y_{i0}) )</td>
<td>( 2(Y_{c0} + Y_{i0}) )</td>
</tr>
<tr>
<td>Type-II</td>
<td>( Y_{a1} - Y_{i2} )</td>
<td>( 2Y_{b1} )</td>
<td>( 2(Y_{c1} + Y_{i2}) )</td>
</tr>
<tr>
<td>Type-III</td>
<td>( Y_{a2} - Y_{i3} + Y_{o3} )</td>
<td>( \frac{(Y_{a0} + Y_{o3})^2}{2(Y_{b0} - Y_{o3})} )</td>
<td>( Y_{a2} - Y_{i3} + 2Y_{i3} + 2Y_{o3} )</td>
</tr>
</tbody>
</table>

Baluns are given in Table I. It should be noted that the output resistance \( R_{out} \) in this equivalent circuit is one fourth of the resistance \( R_{bal} \) at the balanced output of the balun. All the transmission-line sections must also be commensurate, i.e., \( \lambda/4 \) at the design frequency.

The general form of \( n \)-th order Chebyshev insertion loss function for the networks shown in Fig. 6 is given by

\[
L = 1 + K^2 + \epsilon^2 \left[ \frac{1 + \sin \theta_c T_n \left( \frac{\cos \theta}{\cos \theta_c} \right) - (1 - \sin \theta_c) T_n - 2 \left( \frac{\cos \theta}{\cos \theta_c} \right)}{2 \sin \theta} \right]^2 \tag{16}
\]

where \( T_n \) is the Chebyshev polynomial of the first kind of degree \( n \), and \( \theta \) and \( \theta_c \) are the electrical length of the transmission lines and that at the cutoff frequency, respectively. The minimum and maximum of \( L \) in the passband are \( 1 + K^2 \) and \( 1 + K^2 + \epsilon^2 \), respectively. By applying the Richards’ transformation \( s = j \tan \theta \) to (16), the squared modulus reflection coefficient can be expressed by

\[
|S_{11}(s)|^2 = \frac{|S_{22}(s)|^2}{L - \epsilon^2} = \frac{a_n s^{2n} + a_{n-1} s^{2(n-1)} + \cdots + a_1}{b_n s^{2n} + b_{n-1} s^{2(n-1)} + \cdots + b_1} + \frac{N(s)N(-s)}{D(s)D(-s)} \tag{17}
\]

where the coefficient \( a_i \) and \( b_i \) for \( n = 2 \) and \( 3 \) can be found in [14], and those for \( n = 4 \) are given in the paper. They are all summarized in the Appendix.

The zeros and poles of (17) need to be solved to determine its constituents \( N(s) \) and \( D(s) \). In order to have realizable networks, the denominator of \( S_{11}(s) \), \( D(s) \), should be a strictly Hurwitz polynomial, i.e., all the left-hand-side poles in the \( s \)-plane are chosen to determine \( D(s) \). However, the numerator \( N(s) \) is not necessary to be Hurwitz, and the zeros can be selected in conjugate pairs in either the left- or right-hand side of the \( s \) plane. Therefore, the input reflection coefficient \( S_{11}(s) \) can be written in the form of

\[
S_{11}(s) = \frac{N(s)}{D(s)} = C \prod_j \frac{s - z_j}{s - p_j} \tag{18}
\]

where \( z_i \) is the selected zero and \( p_j \) is the left-hand-side pole in conjugate pairs for \( S_{11}(s) \), and \( C \) is a constant to ensure \( S_{11}(0) = -1 \). Generally, there are two choices of \( S_{11}(s) \) for \( n = 2 \) and four for \( n = 3 \) and \( 4 \) with a given Chebyshev insertion loss function. For the case of \( S_{11}(s) \) having perfect matches in the passband, i.e., \( K = 0 \), all zeros will lie on the \( j\Omega \) axis, thus, there are no alternative choices. Therefore, the expressions of \( N(s) \) for \( n = 2 \) and \( 3 \) in [14] are not complete since Hurwitz polynomials are assumed. The output reflection coefficient can also be found as

\[
S_{22}(s) = \frac{N(-s)}{D(s)}. \tag{19}
\]

The input and output admittances can be calculated by

\[
Y_{in}(s) = \frac{1}{R_1} \frac{1 - S_{11}(s)}{1 + S_{11}(s)} \tag{20}
\]

\[
Y_{out}(s) = \frac{1}{R_2} \frac{1 - S_{22}(s)}{1 + S_{22}(s)} \tag{21}
\]

The characteristic admittance of each transmission-line section of the equivalent circuit can then be evaluated with the derived \( Y_{in} \) and \( Y_{out} \). The details are discussed as follows.

A. \( n = 2 \)

The input admittance of the circuit shown in Fig. 6(a) in terms of the characteristic admittances of the transmission-line sections can be expressed by

\[
Y_{in}(s) = \frac{R_2 Y_B^2 s^2 + (Y_A + Y_B) s + R_0 (Y_A Y_B + Y_A Y_C + Y_B Y_C)}{s^2 + R_2 (Y_B + Y_C) s} = \frac{n_2 s^2 + n_1 s + n_0}{d_2 s^2 + d_1 s}. \tag{22}
\]

Comparison between the corresponding coefficients gives the characteristic admittances as

\[
Y_B = \sqrt{\frac{n_2}{d_2 R_2}} \tag{23}
\]

\[
Y_A = \frac{n_1}{d_2} - \sqrt{\frac{n_2}{d_2 R_2}} \tag{24}
\]

\[
Y_C = \frac{d_1}{d_2 R_2} - \sqrt{\frac{n_2}{d_2 R_2}}. \tag{25}
\]

B. \( n = 3 \ (case \ a) \)

For the third-order balun design with a transmission line at the input (case a), as shown in Fig. 6(b), the input unit
element can be extracted by applying Richards’ theorem to $Y_i(s)$, i.e.,

$$Y_i = Y_{in}(1). \quad (26)$$

The input admittance of the remaining network is then solved by removing the unit element as

$$Y'_{in}(s) = Y_i \frac{Y_{in}(s) - Y_{in}(s)}{Y_{in}(s)} - Y_i. \quad (27)$$

Although the numerator and denominator of $Y'_{in}(s)$ appear to be one degree higher than those of $Y_{in}(s)$, they have a common factor $s^{-2}$. Therefore, $Y'_{in}(s)$ is reduced to the formation given in (22). The characteristic admittances $Y_A$, $Y_B$, and $Y_C$ are then obtained accordingly.

C. $n = 3 \ (case \ b)$

The equivalent circuit of the third-order balun design in case b shown in Fig. 6(c) is similar to that of case a, whereas the unit elements and short-circuited stubs are presented in reverse order. Therefore, the output transmission line can be derived by

$$Y_0 = Y_{out}(1). \quad (28)$$

The output admittance of the rest of the circuit are then calculated by

$$Y'_{out}(s) = Y_o \frac{Y_{out}(s) - Y_{out}(s)}{Y_{out}(s)} - Y_o. \quad (29)$$

which can be used to obtain $Y_A$, $Y_B$, and $Y_C$.

D. $n = 4$

The balun design with fourth-order response is shown in Fig. 6(d), where additional unit elements are connected to both the input and output of the three-line balun. Similarly, applying (26) and (28) gives the characteristic admittances of these two unit elements. After removing the input unit element using (27), the input admittance of the rest of the circuit is obtained
and can be expressed by (30), shown at the bottom of this page. Comparing the coefficients in (30) gives the solutions for the characteristic admittances of the rest of the transmission-line sections as follows:

\[ Y_B = \sqrt{\frac{R_2 Y_0^2 n_3}{d_3}} \]  
\[ Y_A = \frac{n_2}{d_3} - \sqrt{\frac{R_2 Y_0^2 n_3}{d_3} - \frac{R_A Y_0 n_3}{d_3}} \]  
\[ Y_C = \frac{Y_0 d_1}{d_3} - \sqrt{\frac{R_2 Y_0^2 n_3}{d_3}} \]  

For convenience, the circuit parameters of balun designs for \( n = 2 \) to \( n = 4 \) with \( R_1 = 50 \Omega \) and \( R_2 = 25 \Omega \) are calculated and summarized in Tables II and III. The baluns in Table II have \( \vert S_{11} \vert_{\text{max}} = -20 \text{ dB} \) and \( \vert S_{11} \vert_{\text{min}} = -25 \text{ dB} \) in the passband, and those in Table III have \( \vert S_{11} \vert_{\text{max}} = -20 \text{ dB} \) and \( \vert S_{11} \vert_{\text{min}} = -\infty \text{ dB} \). For the balun designs with other values of \( R_1 \) and \( R_2 \), the parameters can also be derived from these tables. By adding redundancy unit elements to the input and output, the circuits in Fig. 6(a)–(c) can all be put into the same form, as shown in Fig. 6(d). It is then transformed to its equivalent circuit as shown in Fig. 7 by using Kuroda’s identity of the second kind. Redundancy unit elements at the two ends, if they exist, can now be removed. This completes the impedance transformation. For example, a fourth-order balun with \( R_1 = 50 \Omega \) and \( R_2 = 25 \Omega \) is designed to have a fractional bandwidth of 120% and \( \vert S_{11} \vert_{\text{max}} = -20 \text{ dB} \) and \( \vert S_{11} \vert_{\text{min}} = -\infty \text{ dB} \) in the passband, the admittances would be \( Y_A = 0.00325 \Omega \), \( Y_{12} = 0.00325 \Omega \), \( Y_{13} = 0.04374 \Omega \), \( Y_C = 0.03724 \Omega \), and \( Y_D = 0.05230 \Omega \). If a balun design with \( R_1 = 100 \Omega \) and \( R_2 = 12.5 \Omega \) is needed, the corresponding load impedance ratios \( r_1 \) and \( r_2 \) in Fig. 7 are 2 and 0.5, respectively. The new parameters are calculated as shown in this figure, and the results are \( Y_A = 0.01308 \Omega \), \( Y_{12} = -0.02350 \Omega \), \( Y_{13} = 0.04374 \Omega \), \( Y_C = 0.11822 \Omega \), and \( Y_D = 0.1040 \Omega \).

The characteristic admittances \( Y_A \), \( Y_B \), and \( Y_C \) are now known and they must be realized by three-coupled lines. In order to obtain a realizable circuit, the characteristic admittances for type-I balun should follow the following criteria:

\[ Y_B = 2\nu(C_{11} - C_{12}) > 0 \]
\[ Y_C = 2\nu(C_{12} + C_{13}) > 0 \]
\[ Y_A + \frac{Y_B}{2} = 2\nu(C_{22} - C_{12}) = \nu(C_{20} + C_{23}) \geq 0 \]
\[ Y_A + \frac{Y_C}{2} = 2\nu(C_{22} - C_{23}) = \nu(C_{20} + C_{12}) \geq 0 \]  

The criteria can also be found as

\[ Y_B = 2\nu C_{23} > 0 \]
\[ Y_A + \frac{Y_B}{2} = \nu C_{12} \geq 0 \]
\[ Y_C - Y_B = 2\nu(C_{30} + 2C_{13}) \geq 0 \]  

for the type-II balun and

\[ Y_C - Y_B = 2\nu(C_{11} + C_{13} + C_{12} - C_{23}) \]
\[ = 2\nu(C_{33} + C_{12} + C_{13} - C_{23}) \]
\[ = 2\nu(C_{30} + 2C_{13} + C_{12}) \geq 0 \]  

for the type-III balun. For example, \( Y_C - Y_B < 0 \) is found for the above balun design with \( R_1 = 50 \Omega \) and \( R_2 = 25 \Omega \) and leads to unrealistic three-coupled lines for type-II and type-III baluns. Moreover, type-I and type-II baluns are not applicable
to the balun design with $R_1 = 100 \, \Omega$ and $R_2 = 125 \, \Omega$ since $Y_A + Y_B/2 < 0$ is obtained. Therefore, the derived admittances in Tables II and III should be carefully checked with the criteria (34)–(36) for realization by different types of three-line baluns.

The admittances $Y_A$, $Y_B$, and $Y_C$ represent different characteristics of the three-coupled lines of these three types of baluns. As examples, $Y_A$ in type-I and type-II baluns can be found as

$$Y_A = Y_{22} - Y_{11} = Y_{22} - Y_{12} - Y_{23} - Y_{13} = \nu(C_{20} - C_{13}) \quad (37)$$

and

$$Y_A = Y_{12} - Y_{23} = \nu(C_{12} - C_{23}) \quad (38)$$

respectively, and is not restricted to be positive. The condition of $Y_A < 0$ for the type-I balun usually means that the balun has a high impedance of line 2 or tight coupling between line 1 and line 3. To ease the implementation, a unit element with characteristic admittance of $Y_L$ added at the input is helpful. It is observed that adding the unit element with characteristic admittance of $Y_L$ at the output would increase $Y_C$, which represents the coupling between line 1 and line 2, and line 1 and line 3 for type-I balun and the admittance of $Y_{33}$ for type-II balun. The characteristic admittance $Y_B$ shows the coupling between line 2 and line 3, and line 1 and line 3 for the type-I balun, the coupling between line 2 and line 3 for the type-II balun, and the difference between the coupling of line 1 and 2 and line 2 and 3 for the type-III balun, and will also be increased due to the presence of the unit element at either the input or the output. Baluns with the same amplitude response could have more than one realization since the selections of zeros of $S_{11}(s)$ are not unique. It is found that the determination of $S_{11}(s)$ with all the left-hand-side zeros in the $s$-plane will lead to the highest $Y_A$ and lowest $Y_C$, and the results are contrary if all the right-hand-side zeros are selected.

IV. DESIGN EXAMPLE

A third-order balun (case a) with 50:100-$\Omega$ terminations from the unbalanced to the balanced port is presented as an example, and the corresponding terminations in the equivalent circuit shown in Fig. 6(b) are $R_1 = 50 \, \Omega$ and $R_2 = 25 \, \Omega$. The passband central frequency is 2 GHz with a fractional bandwidth of 80%, which yields $\theta_0 = 0.3\pi$, and the maximum and minimum of $|S_{11}|$ in the passband are $-20 \, \mathrm{dB}$ and $-25 \, \mathrm{dB}$, respectively. $K^2$ and $e^2$ are then calculated to be 0.00317 and 0.00693, respectively. $|S_{11}|^2$ can now be evaluated using (A3)–(A6) and is given by

$$|S_{11}|^2 = \frac{0.00317s^6 + 0.16469s^4 - 0.86021s^2 + 1.17768}{1.00317s^6 - 2.16459s^4 + 1.03908s^2 - 1.17768} \quad (39)$$

Solving (39) gives the zeros of

$$z_1, z_2 = \pm 7.53484$$

$$z_3, z_3^*, z_4, z_4^* = \left(\pm 0.23005 \pm j1.85158\right) \quad (40)$$

and poles of

$$p_1, p_2 = \pm 1.52196$$

$$p_3, p_3^*, p_4, p_4^* = \left(\pm 0.50240 \pm j0.62808\right). \quad (41)$$

All the zeros and poles in the left-hand side of the $s$-plane are chosen for $S_{11}(s)$ in this example. After applying (18) and (20), the input admittance becomes

$$Y_{in} = \frac{0.02(1.05623s^3 + 3.09703s^2 + 2.76763s + 2.16008)}{0.94377s^3 + 2.19650s^2 + 2.07998s} \quad (42)$$

The characteristic admittance of the input unit element is calculated by employing Richards’s theorem as (26)

$$Y_i = 0.03482 \, \Omega \quad (43)$$

The input admittance of the network after the removal of the input unit element can be derived by (27), and the result is

$$Y_{in}' = \frac{0.05416(s^2 - 1)(s^2 + 1.69451s + 1.31892)}{(s^2 - 1)(s^2 + 1.37662s)} \quad (44)$$

The common factor $s^2 - 1$ in (44) is then cancelled, and the characteristic admittances of the remaining transmission-line sections are obtained by (23)–(25) as follows:

$$Y_B = 0.04654 \, \Omega \quad Y_A = 0.04469 \, \Omega \quad Y_C = 0.00852 \, \Omega \quad (45)$$

These results can, of course, be read out directly from Table II. Apparently, the condition of $Y_C - Y_B < 0$ is obtained in this design and its implementation using type-II and type-III baluns is not possible. Therefore, this design is realized by the type-I
Fig. 9. (a) Simulated and (b) measured response of the balun design. (c) Differences of the transmission magnitude and phase.

The balun is implemented using a multilayer Rogers RO3003 substrate with a total thickness of 90 mil, a dielectric constant of 3, and a loss tangent of 0.0013. The circuit layout and dimensions of the three-line balun are shown in Fig. 8, where line 1 and line 2 of the balun are fabricated on the middle layer, and line 3 is on the upper layer. The thickness between these two layers is 10 mil. Fig. 9(a) and (b) show the simulated and measured reflection and transmission coefficients, respectively. In Fig. 9(a), the electromagnetic simulation results are compared to the ideal response and it shows only a minor deviation in the bandwidth. The measured results shown in Fig. 9(b) have approximately 3.6-dB transmission loss around the central frequency. The simulated and measured imbalance of the magnitude and phase is given in Fig. 9(c), where the measured magnitude imbalance is within 0.6 dB in the passband, and the phase error is less than 4°.

V. Conclusion

The design procedure of three types of three-line baluns has been studied in this paper. The equivalent circuits are derived by using the model of coupled transmission lines. It is found that the equivalent circuits of these three types of baluns have the same network configuration. Therefore, the syntheses for these three types of baluns are the same and their design methods are similar. The synthesis procedure has been derived for the baluns with second- to fourth-order Chebyshev response. Some design guidelines are also presented. A third-order balun design example is used to demonstrate the proposed method, and the measurement results are in good agreement with the theoretical study.
APPENDIX

By observing (16) and (17), the expressions of the numerator and denominator of $|S_{11}(s)|^2$ are almost the same, except the constants $K^2$ and $1 + K^2$. Therefore, only the numerators of $|S_{11}(s)|^2$ for $n = 2$ to 4 are given. The denominators can be derived by replacing $K^2$ in the numerators by $1 + K^2$.

$n = 2$: The numerator of (17) is

$$N(s)N(-s) = (K^2 + \varepsilon^2)s^4 + (2\varepsilon^2 - K^2)s^2 + \beta^2\varepsilon^2$$  (A1)

where

$$\beta = \tan^2 \theta_c + \frac{\tan \theta_c}{\cos \theta_c}. \quad \text{(A2)}$$

$n = 3$: The numerator of (17) for $n = 3$ is given by

$$N(s)N(-s) = K^2s^6 + ps^4 + qs^2 + r$$  (A3)

where

$$p = -2K^2 - \varepsilon^2 \left( \frac{1}{\cos \theta_c} \right)^2 \quad \text{(A4)}$$

$$q = K^2 - 2\varepsilon^2 \left( \frac{(1 + \sin \theta_c)}{\cos \theta_c} \right)^2 \quad \text{(A5)}$$

$$r = -\varepsilon^2 \left( \frac{(1 + \sin \theta_c)}{\cos \theta_c} \right)^2 \quad \text{(A6)}$$

$n = 4$: For $n = 4$, the numerator of (17) is

$$N(s)N(-s) = (K^2 + \varepsilon^2)s^8 + w s^6 + x s^4 + y s^2 + z$$  (A7)

where

$$w = -3K^2 + 2\varepsilon^2 \left( -2 + \cos 2\theta_c \frac{5 + 3 \sin \theta_c}{3} \right) \quad \text{(A8)}$$

$$x = 3K^2 + \varepsilon^2 \left( 6 + \cos^2 \theta_c (3 + 2 \sin \theta_c) (1 + 3 \sin \theta_c)^2 \right) \quad \text{(A9)}$$

$$y = -K^2 + 2\varepsilon^2 \sin \theta_c \frac{(3 + 3 \sin \theta_c + 3 \sin^2 \theta_c)}{\left( \cos \frac{\theta_c}{2} - \sin \frac{\theta_c}{2} \right)^6} \quad \text{(A10)}$$

$$z = \frac{\varepsilon^2 \sin^2 \theta_c \frac{\theta_c}{2} + \sin \frac{\theta_c}{2} \left[ \cos \frac{\theta_c}{2} - \sin \frac{\theta_c}{2} \right]^4 \quad \text{(A11)}$$

REFERENCES


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