Delay-Dependent Observer-Based Fuzzy Control for T-S Fuzzy
Systems with Multiple Non-Commensurate Time Delays

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国科會計畫案號: NSC 93-2218-E-270-002.

Abstract

In this paper, the fuzzy systems with multiple non-commensurate time delays are considered. First, the observer-based state feedback fuzzy controllers are proposed. Based on the delay-dependent Lyapunov functional approach, the delay-dependent control design method is derived under which the parallel-distributed fuzzy control can stabilize the whole fuzzy time-delay system asymptotically. Finally, a numerical example is given to illustrate the control design and their effectiveness.

Keyword: Delay-dependent, T-S fuzzy model, linear matrix inequalities.

1. INTRODUCTION

Recently, the task of effectively controlling nonlinear system with time-delays described by T-S fuzzy model has been noticed. For uncertain nonlinear fuzzy system with time delays, [2] discuss the $H_{\infty}$ control design for T-S fuzzy models. In that papers, the concept of the so-called parallel-distributed compensation (PDC) is proposed and the method of LMI's is used. By using Lyapunov-Razumikhin functional approach, the paper [4] proposed the stability and synthesis of nonlinear time-delay systems via linear T-S fuzzy models. The paper [6] derived some global exponential stability conditions and fuzzy controller design methods for free delayed fuzzy systems. The paper [7] investigates the fuzzy control design for fuzzy time-delay system and derives some stabilization conditions in term of Riccati inequality such that the closed loop fuzzy time-delay system is asymptotically stable. By using fuzzy static output feedback control approach, the stabilization problem is considered for nonlinear interval time-delay systems in paper [8]. The
paper [9] used the fuzzy $H_\infty$ output feedback controllers to stabilize the T-S fuzzy systems with multiple time delays. For uncertain discrete-time-delay fuzzy systems, the paper [10] proposed the robust $H_\infty$ output feedback control design methods to stabilize the systems. By using fuzzy observer-based state feedback controller, the problem of the stabilization for the fuzzy systems with time varying delay and parametric uncertainties is considered in the paper [11]. Based on delay dependent Lyapunov functional approach, the delay-dependent guaranteed cost control for T-S fuzzy system with time delays are proposed by the paper [5].

In this paper, we shall investigate the problem of observer-based state feedback fuzzy control design for nonlinear time-delay system which can be presented by Takagi-Sugeno (T-S) fuzzy models with multiple non-commensurate time delays, and derive some stabilizable criteria such that the closed loop fuzzy time-delay system is asymptotically stable. Moreover, this control design approach is dependent on time delays. These sufficient conditions can be easily transformed into the problem of LMI’s by using Schur complement.

II. SYSTEM DESCRIPTION

Consider a nonlinear system with multiple non-commensurate time delays described by the following T-S fuzzy model. This model is composed of $r$ rules, each rule is represented as follows

$R^i$: If $z_i(t)$ is $M_{i1}$ and... and $z_g(t)$ is $M_{iq}$,

Then $\dot{x}(t) = A_i x(t) + \sum_{l=1}^{g} A_{il} x(t - \tau^i_l) + B_i u(t), \quad \text{ (1)}$

$y(t) = C_i x(t),$

where $x(t) = \psi(t), \quad t \in [-\tau_{\text{max}}, 0], \quad A_i, \quad B_i, \quad A_{il}, \quad \text{and} \quad C_i$ are constant matrices with appropriate dimensions; $M_{ij}$ is the fuzzy set; $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^k$ are the state, control, and output vectors, respectively. $z_i(t), z_2(t),..., z_g(t)$ are the premise variables; and $\psi(t)$ is the initial condition of the state. $\tau_{\text{max}}$ is defined the maximum values of the $\tau^i_l$’s.

By using the standard fuzzy inference method, i.e., a singleton fuzzifier, minimum fuzzy inference, and central-average defuzzifier, (1) can be inferred as

$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t)) [A_i x(t) + \sum_{l=1}^{g} A_{il} x(t - \tau^i_l) + B_i u(t)], \quad \text{(2a)}$

$y(t) = \sum_{i=1}^{r} \mu_i(z(t)) C_i x(t), \quad \text{(2b)}$

where $\mu_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{j=1}^{r} \omega_j(z(t))}, \quad \omega_i(z(t)) = \sum_{q=1}^{q} \omega_q(z_q(t))$.

$\min(M_{iq}(z_q(t)))$, $M_{iq}(z_q(t))$ is the membership grade of $z_q(t)$ in $M_{iq}$. It is seen that $\omega_i(z(t)) \geq 0$ and $\sum_{i=1}^{r} \mu_i(z(t)) = 1$.

III. DELAY DEPENDENT STABILIZATION CONTROLLER DESIGN

In practice, all the states of the systems may be immeasurable or useless. Therefore, the observer-based state feedback controller will be proposed to stabilize the time delay systems. The following fuzzy observer rules $i$ is proposed to estimate each state: [1]
The fuzzy observer-based state feedback controller is defined as

\[ \tilde{x}(t) = A\tilde{x}(t) + \sum_{i=1}^{n} A_{di}\tilde{x}(t-t_i^l) + B_i u(t) - \Xi \{\chi(t) - \tilde{y}(t)\} \]  

(3)

where \( \tilde{x}(t) \in \mathbb{R}^n \) is the observer state of \( x(t) \), \( \Xi \) is the fuzzy observer gain for the \( i \)th observer rule, and

\[ \tilde{y}(t) = \sum_{i=1}^{r} \mu_i C_i \tilde{x}(t) \]  

The overall fuzzy observer is represented as follows:

\[ \tilde{x}(t) = \sum_{i=1}^{r} \mu_i \left\{ A_i \tilde{x}(t) + \sum_{i=1}^{n} A_{di}\tilde{x}(t-t_i^l) + B_i u(t) - \Xi \{\chi(t) - \tilde{y}(t)\} \right\} \]  

(4)

Define the estimated error \( e(t) \) as

\[ e(t) = x(t) - \tilde{x}(t) \]  

(5)

The fuzzy observer-based state feedback controller is modified as

\[ u(t) = -\sum_{i=1}^{r} \mu_i K_i x(t) = -\sum_{i=1}^{r} \mu_i K_i (x(t) - e(t)) \]  

(6)

Therefore, we can use the following equation to describe the original system (2a) and the error system (5):

\[ \dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \left\{ (A_i - B_i K_j) x(t) + \sum_{i=1}^{n} A_{dij}\tilde{x}(t-t_j^l) + B_i K_j e(t) \right\} \]  

(7a)

\[ \dot{e}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \left\{ A_i e(t) + \Xi \{\chi(t) - \tilde{y}(t)\} + \sum_{i=1}^{n} A_{dij} e(t-t_j^l) \right\} \]  

(7b)

Combining the state and error equation in (7), the closed-loop fuzzy time-delay system can be rewritten as

\[ \dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \left\{ \Xi \tilde{x}(t) + \sum_{i=1}^{n} \hat{A}_{di}\tilde{x}(t-t_i^l) \right\} \]  

(8)

where

\[ \hat{A}_{di} = \left[ \begin{array}{cc} A_{di} & 0 \\ 0 & A_{di}^T \end{array} \right] \]  

and

\[ \Xi = \left[ \begin{array}{ccc} A_i - B_i K_j & B_i K_j & 0 \\ 0 & A_i + \Xi \{\chi(t) - \tilde{y}(t)\} \end{array} \right] \]

Now, our work is to design the controllers such that the whole fuzzy time-delay system is stabilized asymptotically. Before proceeding to the main results, four lemmas must be introduced first.

**Lemma 1 [12]:** Tchebysev’s inequality holds for any matrix \( v_i \in \mathbb{R}^{m \times n} \)

\[ \frac{1}{m} \sum_{i=1}^{m} v_i^T v_i \leq \frac{1}{m} \sum_{i=1}^{m} v_i \leq \frac{1}{m} \sum_{i=1}^{m} v_i \]  

(9)

**Lemma 2 [5]:** For any two matrices \( X_j \) and \( Y_i \) for \( i = 1, 2, \ldots, r \) and \( S > 0 \) with appropriate dimensions, we have

\[ 2 \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j X_i^T S X_j \leq 2 \sum_{i=1}^{r} \mu_i \left(X_i^T S X_i + Y_i^T S Y_i\right) \]  

(10)

\[ 2 \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} \mu_i \mu_j \mu_k X_{ij}^T S Y_{kl} \leq 2 \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} \mu_i \mu_j \mu_k \left(X_{ij}^T S X_{ij} + Y_{ij}^T S Y_{ij}\right) \]  

(11)

**Lemma 3 [3]:** Assume that \( v_1(\cdot) \in \mathbb{R}^n \), \( v_2(\cdot) \in \mathbb{R}^m \), and \( \Theta \in \mathbb{R}^{m \times n} \) are defined on the interval \( \Omega \). Then, for any matrices \( \Phi \in \mathbb{R}^{n \times n} \), \( \Psi \in \mathbb{R}^{m \times m} \), and \( \Xi \in \mathbb{R}^{m \times n} \), the following holds:

\[ -2 \int_{\Omega} \left[v_1^T(\theta) \Theta v_2(\theta) d\theta \right] \leq \int_{\Omega} \left[v_1(\theta) \Phi \Psi^{T} - \Theta \Xi \right] v_2(\theta) d\theta \]

(12)

where

\[ \Phi \Psi^{T} \Xi \geq 0 \]  

**Lemma 4 [5]:** For any two matrices \( X \) and
\( Y = Y^T > 0 \) with appropriate dimensions, we have
\[
X^T Y^{-1} X > X^T + X - Y \tag{13}
\]

In following theorem, the observer-controller design method is proposed by using the observer-based state feedback controller (6).

**Theorem 1:** The fuzzy time-delay system (1) can be stabilized asymptotically by the fuzzy observer-based state feedback controller (6), if there exist symmetric matrices
\[
A_{i1} \quad A_{i2} \quad A_{i3} \quad A_{i4} \quad A_{i5} \quad A_{i6} \quad A_{i7} \quad 0 \quad 0
\]

such that the following LMIs hold.

\[
\begin{align}
& \begin{bmatrix}
\Lambda_1 & \Lambda_2 & A_{ii}^T - R_{11}^i & - R_{12}^i & 0 & 0 & 0 & 0 & 0 \\
\Lambda_3 & \Lambda_4 & A_{ii}^T & - R_{21}^i & - R_{22}^i & A_{ii}^T & 0 & 0 & 0 \\
- R_{31}^i & A_{ii}^T & - R_{32}^i & - P_{11}^i & - P_{12}^i & 0 & \sqrt{n} A_{ii}^T & 0 & 0 \\
0 & \Lambda_6 & \sqrt{n} A_{ii}^T & 0 & 0 & 0 & - X_{11}^i & - X_{12}^i & 0 \\
0 & \Lambda_7 & 0 & \sqrt{n} A_{ii}^T & 0 & 0 & - X_{21}^i & - X_{22}^i & 0 \\
\end{bmatrix} < 0 \quad \text{for all } i \text{ and } j \tag{14c}
\end{align}
\]

where \( \tilde{\tau} = \sum_{i=1}^{n} \tilde{\tau}_i \), \( \tilde{\tau}_i = \max_{j} \tau_{ij} \), \( \Lambda_1 = \frac{1}{n} \left( A_i^T + A_i - B_i K_j - K_j^T B_i^T \right) + \tau_{ij} R_{11}^i + R_{12}^i + R_{21}^i + R_{22}^i + P_{11}^i \),

\[
\Lambda_2 = \frac{1}{n} B_i K_j + \tau_{ij} R_{12}^i + R_{12}^i + R_{22}^i + P_{12}^i \quad \Lambda_3 = \frac{1}{n} K_j^T B_i + \tau_{ij} R_{12}^i + R_{12}^i + R_{22}^i + P_{12}^i \tag{15a}
\]

\[
\Lambda_4 = \frac{1}{n} \left( A_i^T + A_i + \Theta_i C_j + C_j^T \Theta_i \right) + \tau_{ij} R_{11}^i + R_{12}^i + R_{22}^i + P_{22}^i \quad \Lambda_5 = \frac{1}{n} \left( A_i - B_i K_j \right) \quad \Lambda_6 = \frac{1}{n} B_i K_j \quad \text{and}
\]

\[
\Lambda_7 = \sqrt{\frac{2}{n} (A_i + \Theta_i C_j)} \tag{15b}
\]

**Remark 1:** If all the time-delays \( \tau_{ij} \) are the same for all rules (i.e., \( \tau_{ij} = \tau_i \) for all \( i \neq j \)), the sufficient criteria (14c) can become to two following criteria:

\[
N_i < 0 \quad \text{for all } i, \tag{15a}
\]

\[
N_i + N_j < 0 \quad \text{for all } i < j \tag{15b}
\]

**IV. AN ILLUSTRATIVE EXAMPLE**

**Example 1.**

Consider a fuzzy system with multiple non-commensurate time delays, as follows:...
Rule 1: If \( x_1(t) \) is about 0 and \( x_2(t) \) is about 0,

Then \[
\dot{x}(t) = \begin{bmatrix}
-1.4 & 0.7 \\
1.4 & -0.9
\end{bmatrix} x(t) \begin{bmatrix}
0.2 & 0 \\
0 & 0.2
\end{bmatrix} x(t - \tau_1^1) + \begin{bmatrix}
0.1 & 0 \\
0 & 0.1
\end{bmatrix} x(t - \tau_2^1) + \begin{bmatrix}
0 \\
0.3
\end{bmatrix} u(t).
\]
\[
y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t).
\]

Rule 2: If \( x_1(t) \) is about 0 and \( x_2(t) \) is about \( \pm 1 \),

Then \[
\dot{x}(t) = \begin{bmatrix}
-2 & 0.39 \\
-2.4 & 1.6
\end{bmatrix} x(t) \begin{bmatrix}
0.2 & 0 \\
0 & 0.2
\end{bmatrix} x(t - \tau_1^2) + \begin{bmatrix}
0.1 & 0 \\
0 & 0.1
\end{bmatrix} x(t - \tau_2^2) + \begin{bmatrix}
0 \\
0.36
\end{bmatrix} u(t).
\]
\[
y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t).
\]

Rule 3: If \( x_1(t) \) is about \( \pm 1 \) and \( x_2(t) \) is about 0,

Then \[
\dot{x}(t) = \begin{bmatrix}
-4.5 & 0.3 \\
-3.2 & 1
\end{bmatrix} x(t) \begin{bmatrix}
0.2 & 0 \\
0 & 0.2
\end{bmatrix} x(t - \tau_1^3) + \begin{bmatrix}
0.1 & 0 \\
0 & 0.1
\end{bmatrix} x(t - \tau_2^3) + \begin{bmatrix}
0 \\
0.39
\end{bmatrix} u(t).
\]
\[
y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t).
\]

The membership functions are shown in Fig 1. By using the approaches of Theorem 1 and LMI's tool, then we have the controller gains as follow:

\[
K_1 = \begin{bmatrix} -2.1014 & 9.7351 \end{bmatrix},
K_2 = \begin{bmatrix} -2.1014 & 9.7351 \end{bmatrix},
K_3 = \begin{bmatrix} -2.1014 & 9.7351 \end{bmatrix},
\]

\[
\mathcal{J}_1 = [1.7026 -11.7474]^T,
\mathcal{J}_2 = [1.4836 -13.4103]^T,
\mathcal{J}_3 = [1.4609 -14.4529]^T.
\]

(17)

Fig 2 shows the simulation results of the open-loop system under initial condition \( x(0) \) and time delays \( \tau_1^1 = 0.41, \tau_2^1 = 0.42, \tau_3^1 = 0.43, \tau_1^2 = 0.15, \tau_2^2 = 0.15 \). Fig 3 shows the simulation results of the closed-loop system with the observer-based state feedback controller (17) under initial condition \( y_i(t) = x(0) \) and time delays \( \tau_1^1 = 0.41, \tau_2^1 = 0.42, \tau_3^1 = 0.43, \tau_1^2 = 0.15, \tau_2^2 = 0.15 \). Under initial condition and controllers, the simulation results all show that the closed-loop state responses can always converge to zero. However, the open-loop state responses can not converge to zero.

V. CONCLUSION

First, the control design approach for guaranteeing the stabilizing of the fuzzy time-delay systems has been derived by using the parallel distributed fuzzy observer-based state feedback controller. This design methodology is all dependent of size of the time delays. The suitable control gains can be obtained easily by using LMI's tool. Finally, a numerical example has been provided and the simulation results have showed the effectiveness of the proposed control design method.

REFERENCE


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**Fig. 1** The membership function of $x_1$ and $x_2$

**Fig. 2** Open-loop state responses of the fuzzy time delay system (16).

**Fig. 3** State and estimated error responses with observer-based state feedback controller