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Nonlinear decentralized state feedback controller for uncertain fuzzy time-delay interconnected systems

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Abstract

In this paper, the perturbed continuous-time interconnected system with time-delay is represented by an equivalent Takagi-Sugeno type fuzzy model having r_i rules for each subsystem. Based on Lyapunov stability theorem and Razumikhin theorem, the nonlinear decentralized state feedback fuzzy controllers are proposed to stabilize the whole perturbed fuzzy time-delay interconnected system asymptotically. Under using this nonlinear controller, the design methods only consider the r_i fuzzy rules rather than $r_i \times r_i$ rules for each subsystem, that is, we can remove the negative effect of the coupled terms. Therefore, these approaches can be less conservative. Moreover, if all the time-delays of each subsystem are the same for all rules, we shall propose less conservative criteria. These criteria do not need the solution of a Lyapunov equation or a Riccati equation. We also do not need to find a common positive matrix P to satisfy any inequality. The so-called "matching condition" for the interconnection matrices and perturbations are not needed. Finally, a numerical example is given to illustrate the control design and its effectiveness.

Keywords: Fuzzy interconnected system; T–S fuzzy model; Decentralized control; Razumikhin theorem; Time-delay

1. Introduction

Large-scale interconnected systems can be found in many real-life practical applications such as electric power systems, nuclear reactors, aerospace systems, economic systems, process control systems, computer networks, and urban traffic network, etc. Therefore, many researchers have paid a

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great deal of attention to design the decentralized controller to stabilize the large-scale systems, such as [9,13]. Moreover, there are few studies concerning with the stabilization control for the nonlinear interconnected systems [4,2]. On the other hand, it is well known that time-delay is often encountered in various engineering systems to be controlled [1,8].

In recent years, fuzzy control or fuzzy system has attracted great attention in academic research and industrial application. The stability and stabilizability issues of fuzzy system have been studied by a lot literature [3,5–8,10–12,14,15]. Takagi and Sugeno et al. [10,15] proposed a kind of fuzzy inference system so-called T–S fuzzy model. A H_∞ decentralized fuzzy model reference tracking control design method for nonlinear interconnected systems has been proposed by [11]. Ref. [7] is concerned with the stability problem of fuzzy large-scale systems. Ref. [6] proposed some stabilized control laws to eliminate the coupled models. The paper [3,5] proposed a necessary and sufficient condition for stabilization, the discrete-time fuzzy control system. The paper [12] considers the stabilization for T–S fuzzy time-delay systems by using state feedback controller.

In this paper, the problem of decentralized stabilization of fuzzy time-delay interconnected systems with nonlinear perturbations is considered. We design the nonlinear decentralized state feedback fuzzy controller to stabilize this system robustly. Due to using this nonlinear controller, we can reduce the fuzzy rules' number of each closed-loop subsystem from $r_i \times r_i$ to r_i. In other words, we can eliminate the coupled term (i.e.,$B_i^l K_i^m$ for all $l \neq m$) to relax the stabilized criteria. In the paper [14], the decentralized PDC fuzzy controller is considered to stabilize the same system. However, the design methods of the paper [14] must include the influence of the coupled terms. In general, the coupled terms do create difficulties in design and computations.

Some notations are defined here first. $\|x\|$ means the Euclidean norm of the vector x, $\|A\|$ means the spectral norm of the matrix A. Moreover, the matrix measure of the matrix A is defined as $\mu(A) = \lambda_{\max}((A^{\mathrm{T}} + A)/2)$.

2. System description

Consider a nonlinear perturbed time-delay interconnected system S composed of N subsystems $S_i, i = 1, 2, \ldots, N$. Each rule of the subsystem S_i can be represented by a T–S fuzzy model as follows [11,14,15]:

$$S_i^l : \begin{cases} If\ z_{i1}\ is\ F_{i1}^l\ and ... and\ z_{in}\ is\ F_{in}^l, \\ Then\ \dot{x}_i(t) = A_i^l x_i(t) + B_i^l u_i(t) + \sum_{j=1}^{N} A_{ij}^l x_j(t - \tau_{ij}^l(t)) + f_i^l(x_i(t), t), \end{cases} \tag{1}$$

$i = 1, 2, \ldots, N;\ l = 1, 2, \ldots, r_i$, where $x_i(t) \in R^{n_i}$, $u_i(t) \in R^{m_i}$ are the state and control vectors of the ith subsystem, respectively. F_{iq}^l $(q = 1, 2, \ldots, n)$ and r_i represent the linguist fuzzy variables of the rule l, and the number of the fuzzy rules in subsystem S_i, respectively; and $z_i(t) = [z_{i1}, z_{i2}, \ldots, z_{in}]$ are some measurable premise variables for subsystem S_i. A_i^l and B_i^l denote the system matrix and input matrix with appropriate dimensions, respectively. $\tau_{ij}^l(t) \in R_+$ and $A_{ij}^l \in R^{n_i \times n_j}$ represent the time delay and the interconnection matrix between the ith and the jth subsystems. $x_i(t) = \psi_i(t),\ t \in [-\tau,\ 0],\ \tau_{ij}^l(t) \leqslant \tau$, for $i, j = 1, 2, \ldots, N,\ l = 1, 2, \ldots, r_i$; $\psi_i(t)$ is the initial condition of the state. The vector $f_i^l(x_i(t), t)$

is a nonlinear perturbation. All the subsystems satisfy two assumptions:
(A1) All pairs $(A_i^l,\ B_i^l)$ are controllable.
(A2) For each $f_i^l(x_i(t),t)$, there exists a constant $b_i^l > 0$ such that $\|f_i^l(x_i(t),t)\| \leqslant b_i^l\|x_i(t)\|$.

If we utilize the standard fuzzy inference method, i.e., a singleton fuzzifier, minimum fuzzy inference, and central-average defuzzifier, (1) can be inferred as [11,15,14]:

$$\dot{x}_i(t) = \sum_{l=1}^{r_i} h_i^l(z_{iq}(t)) \left\{ A_i^l x_i(t) + B_i^l u_i(t) + \sum_{j=1}^{N} A_{ij}^l x_j(t - \tau_{ij}^l(t)) + f_i^l(x_i(t),t) \right\}, \tag{2}$$

where

$$h_i^l(z_{iq}(t)) = \frac{w_i^l(z_{iq}(t))}{\sum_{l=1}^{r_i} w_i^l(z_{iq}(t))}, \quad w_i^l(z_{iq}(t)) = \min_q (F_{iq}^l(z_{iq}(t))). \tag{3}$$

$F_{iq}^l(z_{iq}(t))$ is the grade of membership of $z_{iq}(t)$ in F_{iq}^l. It is seen that $w_i^l(z_{iq}(t)) \geqslant 0,\ l = 1,2,\ldots,r_i$, for all t, and $\sum_{l=1}^{r_i} h_i^l(z_{iq}(t)) = 1$.

3. Stabilization of fuzzy time-delay interconnected systems

In this section, the decentralized control scheme and the fuzzy control approach are employed to design the nonlinear controller. Let the ith nonlinear fuzzy control law corresponding to S_i be of the form

$$u_i(t) = \frac{-\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t)\right)\left(\sum_{l=1}^{r_i} h_i^l x_i^{\mathrm{T}}(t) B_i^l K_i^l x_i(t)\right)}{\left(\sum_{l=1}^{r_i} h_i^l x_i^{\mathrm{T}}(t) B_i^l\right)\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t)\right)}, \tag{4}$$

in which, for convenience, we use the briefness notation h_i^l to denote $h_i^l(z_{iq}(t))$ in (3). It is noted that this control law is continuous and can be defined in $x_i(t_0) \neq 0$ such that $\left(\sum_{l=1}^{r_i} h_i^l x_i^{\mathrm{T}}(t_0) B_i^l\right)\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t_0)\right) = 0$. Because of $\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t)\right) \to 0$ (i.e., $u_i(t) \to 0$) when $x_i(t) \to x_i(t_0)\ \forall i$. Due to $\left(\sum_{l=1}^{r_i} h_i^l\ x_i^{\mathrm{T}}(t) B_i^l\right)\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t)\right) = w^{\mathrm{T}}(x_i(t)) \times w(x_i(t)) = \|w(x_i(t))\|^2$, if exists a $x_i(t_0) \neq 0$ such that $\left(\sum_{l=1}^{r_i} h_i^l x_i^{\mathrm{T}}(t_0) B_i^l\right)\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t_0)\right) = \|w(x_i(t_0))\|^2 = 0$ then $\|w(x_i(t_0))\| = 0$ (i.e., $\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t_0)\right) = 0$, $u_i(t_0) = 0$).

From (2) and (4), the closed-loop fuzzy subsystem becomes

$$\dot{x}_i(t) = \sum_{l=1}^{r_i} h_i^l \left\{ A_i^l x_i(t) - B_i^l \frac{\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t)\right)\left(\sum_{l=1}^{r_i} h_i^l x_i^{\mathrm{T}}(t) B_i^l K_i^l x_i(t)\right)}{\left(\sum_{l=1}^{r_i} h_i^l x_i^{\mathrm{T}}(t) B_i^l\right)\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t)\right)} \right.$$
$$\left. + \sum_{j=1}^{N} A_{ij}^l x_j(t - \tau_{ij}^l(t)) + f_i^l(x_i(t), t) \right\}. \tag{5}$$

Before proceeding to the main results, a lemma must be introduced first.

Lemma 1 (*Davison [4]*). *For any two matrices X and Y with appropriate dimension, we have*

$$X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \leqslant \varepsilon X^{\mathrm{T}}X + \varepsilon^{-1}Y^{\mathrm{T}}Y, \tag{6}$$

for any constant $\varepsilon > 0$.

Now, we are in a position to state the first theorem.

Theorem 1. *Consider the perturbed fuzzy time-delay interconnected system S as* (1). *If one selects the controller gain* K_i^l *to satisfy* (7), *then the overall closed-loop fuzzy time-delay system composed of N subsystem* S_i (5), *is stabilized asymptotically by the nonlinear fuzzy controller* (4).

$$\mu_{iM}^{ll} < -\left[\frac{N}{2} + b_i\right] - M_i\hat{\Phi}_i \tag{7a}$$

$$if\ \exists x_i(t_0) \neq 0\ such\ that\ \left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t_0)\right) = 0\ then\ \forall l\ such\ that$$
$$h_i^l \neq 0,\ x_i^{\mathrm{T}}(t_0) B_i^l K_i^l x_i(t_0) = 0. \tag{7b}$$

in which $\mu_{iM}^{ll} = \max_l \mu(G_i^{ll}),\ G_i^{ll} = (A_i^l - B_i^l K_i^l),\ b_i = \max_l b_i^l,\ \hat{\Phi}_i = \max_{j,l} \|A_{ji}^{l}{}^{\mathrm{T}} A_{ji}^l\|,\ M_i = \max_l M_i^l$, *and* M_i^l *is the number of* $A_{ij}^l \neq 0$, *for* $i, j = 1, 2, \ldots, N,\ l = 1, 2, \ldots, r_i$.

Proof. Let the Lyapunov function for the closed-loop system (5) be as follows:

$$V = \sum_{i=1}^{N} \alpha_i x_i^{\mathrm{T}} x_i = \sum_{i=1}^{N} V_i(x_i), \quad i = 1, 2, \ldots, N. \tag{8}$$

where $\alpha_i > 0$ is a constant scalar to be determined later.

Case 1: $\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t_0)\right) \neq 0$

$$\dot{V} = \sum_{i=1}^{N} \{\dot{x}_i^{\mathrm{T}} \alpha_i x_i + x_i^{\mathrm{T}} \alpha_i \dot{x}_i\} \tag{9}$$

$$= \sum_{i=1}^{N}\sum_{l=1}^{r_i} h_i^l \alpha_i \left\{ \left[A_i^l x_i + B_i^l u_i + \sum_{j=1}^{N} A_{ij}^l x_j(t-\tau_{ij}^l(t)) + f_i^l(x_i) \right]^{\mathrm{T}} x_i + x_i^{\mathrm{T}} \left[A_i^l x_i + B_i^l u_i + \sum_{j=1}^{N} A_{ij}^l x_j(t-\tau_{ij}^l(t)) + f_i^l(x_i) \right] \right\}, \tag{10}$$

$$= \sum_{i=1}^{N} \left\{ \sum_{l=1}^{r_i} h_i^l \left[\alpha_i x_i^{\mathrm{T}}(A_i^{l\,\mathrm{T}} + A_i^l)x_i + 2\alpha_i x_i^{\mathrm{T}} \sum_{j=1}^{N} A_{ij}^l x_j(t-\tau_{ij}^l(t)) + 2\alpha_i x_i^{\mathrm{T}} f_i^l(x_i) \right] -2\alpha_i \left(\sum_{l=1}^{r_i} h_i^l x_i^{\mathrm{T}} B_i^l \right) \frac{\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t)\right)\left(\sum_{l=1}^{r_i} h_i^l x_i^{\mathrm{T}}(t) B_i^l K_i^l x_i(t)\right)}{\left(\sum_{l=1}^{r_i} h_i^l x_i^{\mathrm{T}}(t) B_i^l\right)\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t)\right)} \right\}. \tag{11}$$

Letting $G_i^{ll} = (A_i^l - B_i^l K_i^l)$ and using Lemma 1, and assumption (A2), then we have

$$\dot{V} \leqslant \sum_{i=1}^{N} \left\{ \sum_{l=1}^{r_i} h_i^l [\alpha_i x_i^{\mathrm{T}}(G_i^{ll\,\mathrm{T}} + G_i^{ll})x_i + M_i^l \alpha_i^2 x_i^{\mathrm{T}} x_i + 2\alpha_i x_i^{\mathrm{T}} f_i^l(x_i)] + \sum_{l=1}^{r_j} h_j^l \sum_{j=1}^{N} x_i^{\mathrm{T}}(t-\tau_{ji}^l(t)) A_{ji}^{l\,\mathrm{T}} A_{ji}^l x_i(t-\tau_{ji}^l(t)) \right\}. \tag{12}$$

Let $\alpha_i \geqslant \hat{\Phi}_i$, then we have

$$\dot{V} \leqslant \sum_{i=1}^{N} \left\{ \sum_{l=1}^{r_i} h_i^l [2\alpha_i \mu_i(G_i^{ll}) x_i^{\mathrm{T}} x_i + M_i^l \alpha_i^2 x_i^{\mathrm{T}} x_i + 2\alpha_i b_i^l x_i^{\mathrm{T}} x_i] + \sum_{l=1}^{r_j} h_j^l \sum_{j=1}^{N} V_i(x_i(t-\tau_{ji}^l(t))) \right\}. \tag{13}$$

By using Razumikhin theorem, if there exists a real $\delta > 1$ such that

$$V_i[x_i(t-\theta)] < \delta V_i[x_i(t)] \quad \text{for } \theta \in [0, \tau], \tag{14}$$

then

$$\dot{V} < \sum_{i=1}^{N} \left\{ \sum_{l=1}^{r_i} h_i^l [2\alpha_i \mu(G_i^{ll}) + \alpha_i^2 M_i^l + \delta N \alpha_i + 2\alpha_i b_i^l] x_i^{\mathrm{T}} x_i \right\} \tag{15}$$

$$\leqslant \sum_{i=1}^{N} \left\{ \sum_{l=1}^{r_i} h_i^l [2\alpha_i \mu_{iM}^{ll} + \alpha_i^2 M_i + \delta N \alpha_i + 2\alpha_i b_i] x_i^{\mathrm{T}} x_i \right\}. \tag{16}$$

Define $g_i(\alpha_i) = [2\alpha_i \mu_{iM}^{ll} + \alpha_i^2 M_i + \delta\, N\alpha_i + 2\alpha_i b_i]$. Take the derivative of $g_i(\alpha_i)$ with respect to α_i and set it to be zero, then we obtain

$$\alpha_i = \frac{-\left(\mu_{iM}^{ll} + \delta\frac{N}{2} + b_i\right)}{M_i}. \tag{17}$$

It is seen that the value α_i in (17) force $g_i(\alpha_i)$ to be minimum. Therefore

$$\dot{V} \leqslant \sum_{i=1}^{N}\left\{\sum_{l=1}^{r_i} h_i^l \left[\frac{-\left(\mu_{iM}^{ll} + \delta\frac{N}{2} + b_i\right)^2}{M_i}\right] x_i^{\mathrm{T}} x_i\right\} \equiv -\sum_{i=1}^{N} \eta_i x_i^{\mathrm{T}} x_i. \tag{18}$$

Define $\Omega(\delta) = -\left(\mu_{iM}^{ll} + \delta\frac{N}{2} + b_i\right) - M_i\hat{\Phi}_i$. If $\Omega(\delta) > 0$ hold for all i, then $\dot{V}(x(t)) < 0$. Thus, the whole closed-loop fuzzy time-delay interconnected system is asymptotically stable. If (7a) holds, that is, $\Omega(1) > 0$, then by continuity, there is a $\delta = 1 + \sigma$ with $\sigma > 0$ sufficiently small such that $\Omega(\delta) > 0$ for all i. In other words, if (7a) holds, there exist positive constants $\eta_i,\ i = 1, 2, \ldots, N$ to satisfy (18).

Case 2: $\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t_0)\right) = 0$ and $x_i(t_0) \neq 0$.

If $\sum_{i=1}^{N}\left\{\sum_{l=1}^{r_i} h_i^l \left[\alpha_i x_i^{\mathrm{T}}(G_i^{ll\,\mathrm{T}} + G_i^{ll})x_i + 2\alpha_i x_i^{\mathrm{T}} \sum_{j=1}^{N} A_{ij}^l x_j(t - \tau_{ij}^l(t)) + 2\alpha_i x_i^{\mathrm{T}} f_i^l(x_i)\right]\right\} < 0$, then the condition (7b) leads to $\sum_{i=1}^{N}\left\{\sum_{l=1}^{r_i} h_i^l \alpha_i x_i^{\mathrm{T}}(t_0)(A_i^{l\,\mathrm{T}} + A_i^l)x_i(t_0) + 2\alpha_i x_i^{\mathrm{T}}(t_0) \sum_{j=1}^{N} A_{ij}^l x_j(t_0 - \tau_{ij}^l(t_0)) + 2\alpha_i x_i^{\mathrm{T}}(t_0) f_i^l(x_i(x_0))\right\} < 0$ then $\dot{V}(x(t_0)) < 0$. The proof is completed here. □

Remark 1. In Theorem 1, μ_{iM}^{ll} must be negative. It is seen that the allowable perturbation bounds are relevant to the number of $A_{ij}^l \neq 0$ (i.e., $\max_l M_i^l$), the value of $\max_{j,l} \|A_{ji}^{l\,\mathrm{T}} A_{ji}^l\|$, and μ_{iM}^{ll}. Therefore, the system can tolerate a larger perturbation bounds if the value of μ_{iM}^{ll} is more negative.

Remark 2. In Theorem 1 of paper [6], the system must be restricted to SIMO models. But, we can remove this restriction in Theorem 1 of this paper. Next, paper [6] only considers the general T–S fuzzy system without delays and perturbations. However, we consider the T–S fuzzy interconnected system with delays and perturbations.

Remark 3. If system (5) without time delay (i.e., $\tau_{ij}^l(t) = 0$), then the criterion (7a) of Theorem 1 can be simplified to $\mu_{iM}^{ll} < -[\sqrt{M_i\Phi_i} + b_i]$, where $\Phi_i = \max_l \left\|\sum_{j=1}^{N} A_{ji}^{l\,\mathrm{T}} A_{ji}^l\right\|$.

If $B_i^l = g_l B_i$ and $g_l > 0\ \forall i, l$, then the sufficient condition (7b) of Theorem 1 can be removed.

Corollary 1. *Consider the perturbed fuzzy time-delay interconnected system S as* (1) *with* $B_i^l = g_l B_i$. *If one selects the controller gain* K_i^l *to satisfy* (7a), *then the overall closed-loop fuzzy time-delay system composed of N subsystem* S_i (5), *is stabilized asymptotically by the nonlinear fuzzy controller* (4).

Proof. According to (3), we have $h_i^l \geqslant 0$ and $\sum_{l=1}^{r_i} h_i^l = 1$. If $\exists\, x_i(t_0) \neq 0$ such that $\left(\sum_{l=1}^{r_i} h_i^l {B_i^l}^{\mathrm{T}} x_i(t_0)\right) = 0$, then we have $B_i^{\mathrm{T}} x_i(t_0) = 0$ or $x_i^{\mathrm{T}}(t_0) B_i = 0,\ \forall i$. Because of $\left(\sum_{l=1}^{r_i} h_i^l {B_i^l}^{\mathrm{T}} x_i(t_0)\right) = \left(\sum_{l=1}^{r_i} h_i^l g_l B_i^{\mathrm{T}} x_i(t_0)\right) = \left(B_i^{\mathrm{T}} x_i(t_0) \times \sum_{l=1}^{r_i} h_i^l g_l\right)$ and $\sum_{l=1}^{r_i} h_i^l g_l \neq 0$, such that if $\left(\sum_{l=1}^{r_i} h_i^l {B_i^l}^{\mathrm{T}} x_i(t_0)\right) = 0$ then $B_i^{\mathrm{T}} x_i(t_0) = 0$ or $x_i^{\mathrm{T}}(t_0) B_i = 0\ \forall i$. Due to $x_i^{\mathrm{T}}(t_0) B_i^l K_i^l x_i(t_0) = (x_i^{\mathrm{T}}(t_0) B_i) \times (g_l K_i^l x_i(t_0))$, if $B_i^{\mathrm{T}} x_i(t_0) = 0$ or $x_i^{\mathrm{T}}(t_0) B_i = 0$ then $x_i^{\mathrm{T}}(t_0) B_i^l K_i^l x_i(t_0) = 0$. In other words, if $\exists\, x_i(t_0) \neq 0$ such that $\left(\sum_{l=1}^{r_i} h_i^l {B_i^l}^{\mathrm{T}} x_i(t_0)\right) = 0$, then we have $h_i^l \neq 0,\ x_i^{\mathrm{T}}(t_0) B_i^l K_i^l x_i(t_0) = 0$. □

If all the time-delays $\tau_{ij}^l(t)$ are the same for all rules (i.e., $\tau_{ij}^l(t) = \tau_{ij}$ for all l and τ_{ij} is a constant for $i,\ j = 1, 2, \ldots, N$), we shall propose the following theorem which is simpler and less conservative than Theorem 1.

Theorem 2. *Consider the perturbed fuzzy time-delay interconnected system S as* (1) *with* $\tau_{ij}^l(t) = \tau_{ij}$ *for all* l. *If one selects the controller gain* K_i^l *to satisfy* (19), *then the overall closed-loop fuzzy time-delay system composed of N subsystem* S_i (5) *with* $\tau_{ij}^l(t) = \tau_{ij}$, *is stabilized asymptotically by the nonlinear fuzzy controller* (4).

$$\mu_{iM}^{ll} < -\left[\sqrt{M_i \tilde{\Phi}_i} + b_i\right] \tag{19a}$$

$$\begin{aligned} &if\ \exists\, x_i(t_0) \neq 0\ such\ that\ \left(\sum_{l=1}^{r_i} h_i^l {B_i^l}^{\mathrm{T}} x_i(t_0)\right) = 0\ then\ \forall l\ such\ that \\ &\quad h_i^l \neq 0, x_i^{\mathrm{T}}(t_0) B_i^l K_i^l x_i(t_0) = 0. \end{aligned} \tag{19b}$$

in which $\mu_{iM}^{ll} = \max_l \mu(G_i^{ll}),\ b_i = \max_l b_i^l,\ \tilde{\Phi}_i = \left\| \sum_{j=1}^{N} S_{ji} \right\|$, $S_{ji} \geqslant \max_l {A_{ji}^l}^{\mathrm{T}} A_{ji}^l$, *and* $M_i = \max_l M_i^l$, *and* M_i^l *is the number of* $A_{ij}^l \neq 0$, *for* $i,\ j = 1, 2, \ldots, N,\ l = 1, 2, \ldots, r_i$.

Proof. Let the Lyapunov function for the closed-loop system (5) with $\tau_{ij}^l(t) = \tau_{ij}$ be as follows:

$$V = \sum_{i=1}^{N} \left\{ \alpha_i x_i^{\mathrm{T}} x_i + \sum_{j=1}^{N} \int_{t-\tau_{ij}}^{t} x_j^{\mathrm{T}}(\theta) S_{ij} x_j(\theta) \mathrm{d}\theta \right\} = \sum_{i=1}^{N} V_i(x_i), \quad i = 1, 2, \ldots, N, \tag{20}$$

where $\alpha_i > 0$ is a constant scalar to be determined later.

Case 1: $\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t_0)\right) \neq 0$

$$\dot{V} = \sum_{i=1}^{N}\left\{\dot{x}_i^{\mathrm{T}}\alpha_i x_i + x_i^{\mathrm{T}}\alpha_i\dot{x}_i + \sum_{j=1}^{N} x_j^{\mathrm{T}} S_{ij} x_j - \sum_{j=1}^{N} x_j^{\mathrm{T}}(t-\tau_{ij}) S_{ij} x_j(t-\tau_{ij})\right\} \tag{21}$$

$$\begin{aligned} &= \sum_{i=1}^{N}\left\{\sum_{l=1}^{r_i} h_i^l\left[\alpha_i x_i^{\mathrm{T}}(A_i^{l\,\mathrm{T}} + A_i^l)x_i + 2\alpha_i x_i^{\mathrm{T}}\sum_{j=1}^{N} A_{ij}^l x_j(t-\tau_{ij}) + 2\alpha_i x_i^{\mathrm{T}} f_i^l(x_i)\right]\right. \\ &\quad + \sum_{j=1}^{N} x_j^{\mathrm{T}} S_{ij} x_j - 2\alpha_i\left(\sum_{l=1}^{r_i} h_i^l x_i^{\mathrm{T}} B_i^l\right)\frac{\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t)\right)\left(\sum_{l=1}^{r_i} h_i^l x_i^{\mathrm{T}} B_i^l K_i^l x_i(t)\right)}{\left(\sum_{l=1}^{r_i} h_i^l x_i^{\mathrm{T}}(t) B_i^l\right)\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t)\right)} \\ &\quad \left. - \sum_{j=1}^{N} x_j^{\mathrm{T}}(t-\tau_{ij}) S_{ij} x_j(t-\tau_{ij})\right\}. \end{aligned} \tag{22}$$

Letting $G_i^{ll} = (A_i^l - B_i^l K_i^l)$ and using Lemma 1 and assumption (A2), then we have

$$\begin{aligned} \dot{V} \leqslant \sum_{i=1}^{N} &\left\{\sum_{l=1}^{r_i} h_i^l[2\alpha_i\mu_i(G_i^{ll})x_i^{\mathrm{T}}x_i + M_i^l\alpha_i^2 x_i^{\mathrm{T}}x_i + 2\alpha_i b_i^l x_i^{\mathrm{T}}x_i]\right. \\ &+ \sum_{l=1}^{r_j} h_j^l \sum_{j=1}^{N} x_i^{\mathrm{T}}(t-\tau_{ji}) A_{ji}^{l\,\mathrm{T}} A_{ji}^l x_i(t-\tau_{ji}) \\ &\left. + \sum_{l=1}^{r_j} h_j^l\left[x_i^{\mathrm{T}}\sum_{j=1}^{N} S_{ji}x_i - \sum_{j=1}^{N} x_i^{\mathrm{T}}(t-\tau_{ji}) S_{ji} x_i(t-\tau_{ji})\right]\right\}. \end{aligned} \tag{23}$$

Let $S_{ji} \geqslant \max_l A_{ji}^{l\,\mathrm{T}} A_{ji}^l$, then we have

$$\dot{V} \leqslant \sum_{i=1}^{N}\left\{\sum_{l=1}^{r_i} h_i^l[2\alpha_i\mu_i(G_i^{ll})x_i^{\mathrm{T}}x_i + M_i\alpha_i^2 x_i^{\mathrm{T}}x_i + \tilde{\Phi}_i x_i^{\mathrm{T}}x_i + 2\alpha_i b_i^l x_i^{\mathrm{T}}x_i]\right\} \tag{24}$$

$$\leqslant \sum_{i=1}^{N}\left\{\sum_{l=1}^{r_i} h_i^l[2\alpha_i\mu_{iM}^{ll} + \alpha_i^2 M_i + \tilde{\Phi}_i + 2\alpha_i b_i]x_i^{\mathrm{T}}x_i\right\}. \tag{25}$$

Define $g_i(\alpha_i) = [2\alpha_i\mu_{iM}^{ll} + \alpha_i^2 M_i + \tilde{\Phi}_i + 2\alpha_i b_i]$. Take the derivative of $g_i(\alpha_i)$ with respect to α_i and set it to be zero, then we obtain

$$\alpha_i = \frac{-(\mu_{iM}^{ll} + b_i)}{M_i}. \tag{26}$$

It is seen that the value α_i in (26) force $g_i(\alpha_i)$ to be minimum. Therefore

$$\dot{V} \leqslant \sum_{i=1}^{N} \left\{ \sum_{l=1}^{r_i} h_i^l \left[\frac{-(\mu_{iM}^{ll} + b_i)^2 + M_i \tilde{\Phi}_i}{M_i} \right] x_i^{\mathrm{T}} x_i \right\} \equiv - \sum_{i=1}^{N} \eta_i x_i^{\mathrm{T}} x_i. \tag{27}$$

If (19a) holds, there exist positive constants $\eta_i, \; i = 1, 2, \ldots, N$ to satisfy (27).

Case 2: $\left(\sum_{l=1}^{r_i} h_i^l B_i^{l\,\mathrm{T}} x_i(t_0) \right) = 0$ and $x_i(t_0) \neq 0$.

The proof of case 2 is similar to that of Theorem 1. Then, the proof is completed here. □

If $B_i^l = g_l B_i$ and $g_l > 0 \; \forall i, l$, then the sufficient condition (19b) of Theorem 2 can be removed.

Corollary 2. *Consider the perturbed fuzzy time-delay interconnected system S as* (1) *with* $B_i^l = g_l B_i$ *and* $\tau_{ij}^l(t) = \tau_{ij}$. *If one selects the controller gain* K_i^l *to satisfy* (19a), *then the overall closed-loop fuzzy time-delay system composed of N subsystem* S_i (5) *with* $B_i^l = g_l B_i$ *and* $\tau_{ij}^l(t) = \tau_{ij}$, *is stabilized asymptotically by the nonlinear fuzzy controller* (4).

Remark 4. It is obviously, the criteria of Theorems 1, 2 of paper [14] are more conservative and complex than the criterion (19a). Next, the methods [14] also need many computations because they must include the negative effect of the coupled terms (i.e., $\mu_{iM}^{lm}, \; \forall l < m$). However, the criterion (19b) also brings another restriction. But, Corollary 2 shows that these restrictions can be removed if $B_i^l = g_l B_i$. This assumption $B_i^l = g_l B_i$ can find some practice systems, such as [11,16].

4. An illustrative example and simulation

Example. Consider an interconnected system S composed of three fuzzy subsystems S_i as

Subsystem 1:

Rule 1: *If* $x_{11}(t)$ *is about* 0 *and* $x_{12}(t)$ *is about* 0,

$$Then\ \dot{x}_1(t) = \begin{bmatrix} -3 & 8 \\ 2 & -5 \end{bmatrix} x_1(t) + \begin{bmatrix} 0.8 \\ 0.8 \end{bmatrix} u_1 + \sum_{j \neq 1}^{3} A_{1j}^1 x_j(t - \tau_{1j}^1(t)) + f_1^1(x_1(t), t).$$

Rule 2: *If* $x_{11}(t)$ *is about* 0 *and* $x_{12}(t)$ *is about* ± 1,

$$Then\ \dot{x}_1(t) = \begin{bmatrix} -4 & 5 \\ 3 & -3 \end{bmatrix} x_1(t) + \begin{bmatrix} 0.9 \\ 0.9 \end{bmatrix} u_1 + \sum_{j \neq 1}^{3} A_{1j}^2 (t - \tau_{1j}^2(t)) + f_1^2(x_1(t), t).$$

Rule 3: *If* $x_{11}(t)$ *is about* ± 1 *and* $x_{12}(t)$ *is about* 0,

$$Then\ \dot{x}_1(t) = \begin{bmatrix} -5 & 6 \\ 2 & -2 \end{bmatrix} x_1(t) + \begin{bmatrix} 0.85 \\ 0.85 \end{bmatrix} u_1 + \sum_{j \neq 1}^{3} A_{1j}^3 x_j(t - \tau_{1j}^3(t)) + f_1^3(x_1(t), t). \tag{28}$$

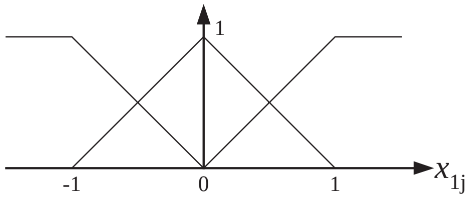

Fig. 1. The membership function of x_{11} and x_{12}.

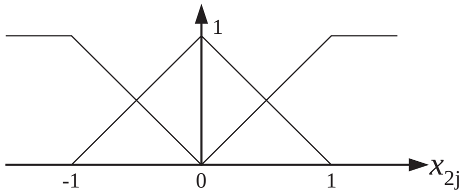

Fig. 2. The membership function of x_{21} and x_{22}.

Subsystem 2:
Rule 1: *If* $x_{21}(t)$ *is about* 0 *and* $x_{22}(t)$ *is about* 0,

$$Then\ \dot{x}_2(t) = \begin{bmatrix} -2.25 & 6 \\ 1.5 & -3.75 \end{bmatrix} x_2(t) + \begin{bmatrix} 1.05 \\ 1.05 \end{bmatrix} u_2 + \sum_{j \neq 2}^{3} A_{2j}^1 x_j(t - \tau_{2j}^1(t)) + f_2^1(x_2(t), t).$$

Rule 2: *If* $x_{21}(t)$ *is about* ± 1 *and* $x_{22}(t)$ *is about* -1,

$$Then\ \dot{x}_2(t) = \begin{bmatrix} -3.75 & 4.5 \\ 1.5 & -1.5 \end{bmatrix} x_2(t) + \begin{bmatrix} 0.97 \\ 0.97 \end{bmatrix} u_2 + \sum_{j \neq 2}^{3} A_{2j}^2 x_j(t - \tau_{2j}^2(t)) + f_2^2(x_2(t), t). \quad (29)$$

Subsystem 3:
Rule 1: *If* $x_{31}(t)$ *is about* 0 *and* $x_{32}(t)$ *is about* 0,

$$Then\ \dot{x}_3(t) = \begin{bmatrix} -4.8 & 6 \\ 3.6 & -3.6 \end{bmatrix} x_3(t) + \begin{bmatrix} 0.96 \\ 0.96 \end{bmatrix} u_3 + \sum_{j \neq 3}^{3} A_{3j}^1 x_j(t - \tau_{3j}^1(t)) + f_3^1(x_3(t), t).$$

Rule 2: *If* $x_{31}(t)$ *is about* ± 2 *and* $x_{32}(t)$ *is about* -1,

$$Then\ \dot{x}_3(t) = \begin{bmatrix} -6 & 7.2 \\ 2.4 & -2.4 \end{bmatrix} x_3(t) + \begin{bmatrix} 1.02 \\ 1.02 \end{bmatrix} u_3 + \sum_{j \neq 3}^{3} A_{3j}^2 x_j(t - \tau_{3j}^2(t)) + f_3^2(x_3(t), t). \quad (30)$$

The membership functions for $x_{11},\ x_{12},\ x_{21},\ x_{22},\ x_{31}$, and x_{32} are shown in Figs. 1–3, respectively. Moreover, the interconnections among three subsystems are given as

$$A_{12}^1 = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.1 \end{bmatrix}, \quad A_{13}^1 = \begin{bmatrix} 0 & 0.5 \\ 0 & 0.25 \end{bmatrix}, \quad A_{12}^2 = \begin{bmatrix} 0.25 & 0.5 \\ 0.25 & 0.5 \end{bmatrix}, \quad A_{13}^2 = \begin{bmatrix} 0.3 & 0.3 \\ 0.6 & 0 \end{bmatrix}$$

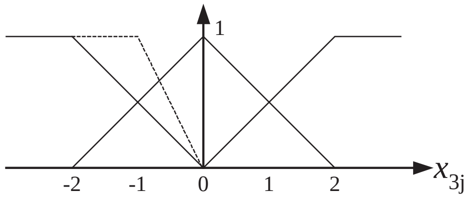

Fig. 3. The membership function of x_{31} and x_{32}.

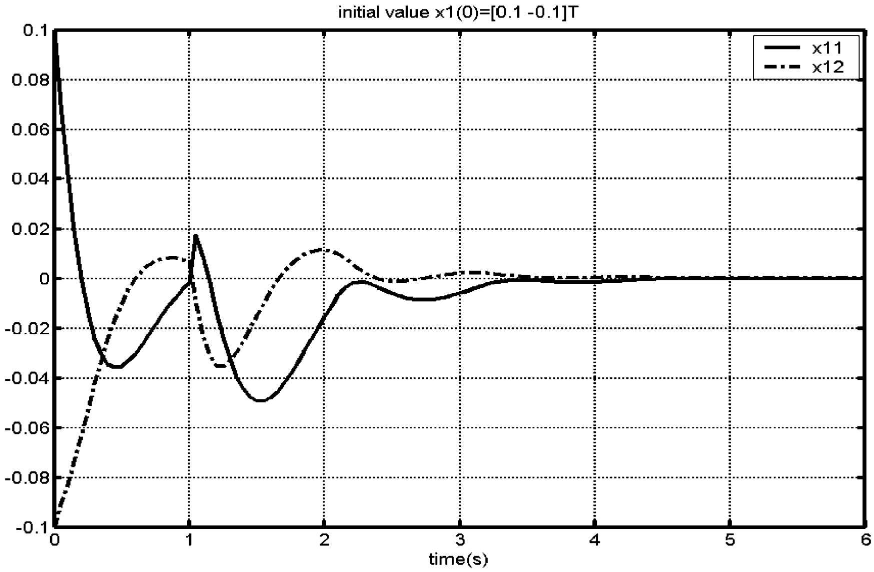

Fig. 4. The state responses of x_{11} and x_{12}.

$$A_{12}^3 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.18 \end{bmatrix}, \quad A_{13}^3 = \begin{bmatrix} 0.27 & 0.18 \\ 0.27 & 0.27 \end{bmatrix}, \quad A_{21}^1 = \begin{bmatrix} 0.225 & 0 \\ 0.45 & 0.225 \end{bmatrix}, \quad A_{23}^1 = \begin{bmatrix} 0.45 & 0.225 \\ 0 & 0.225 \end{bmatrix}$$

$$A_{21}^2 = \begin{bmatrix} 0 & 0.45 \\ 0 & 0.225 \end{bmatrix}, \quad A_{23}^2 = \begin{bmatrix} 0.45 & 0.45 \\ 0.45 & 0 \end{bmatrix}, \quad A_{31}^1 = \begin{bmatrix} 0.54 & 0 \\ 0 & 0.324 \end{bmatrix}, \quad A_{32}^1 = \begin{bmatrix} 0 & 0.08 \\ 0 & 0.54 \end{bmatrix}$$

$$A_{31}^2 = \begin{bmatrix} 0.54 & 0.54 \\ 0.08 & 0 \end{bmatrix}, \quad A_{32}^2 = \begin{bmatrix} 0.54 & 0.08 \\ 0.54 & 0.08 \end{bmatrix}$$

(i.e., $M_i^l = 2\ \forall i, l$). Moreover, $f_i^l(x_i(t), t) = \begin{bmatrix} b_i^l x_{i1}(t) \sin\left(x_{i1}(t) + \frac{\pi}{2}\right) \\ b_i^l x_{i2}(t) \sin\left(x_{i2}(t) + \frac{\pi}{2}\right) \end{bmatrix}$ satisfies (A2) for all i, l.

We choose the controller gain matrices K_i^l for this system as follows:

$$K_1^1 = [5.25\ \ 3.75], \quad K_1^2 = [6.2875\ \ 8.625], \quad K_1^3 = [4.125\ \ 6.25], \quad K_2^1 = [7.35\ \ 5.25],$$
$$K_2^2 = [5.775\ \ 8.75], \quad K_3^1 = [4.024\ \ 5.52], \quad K_3^2 = [2.64\ \ 4.0]. \tag{31}$$

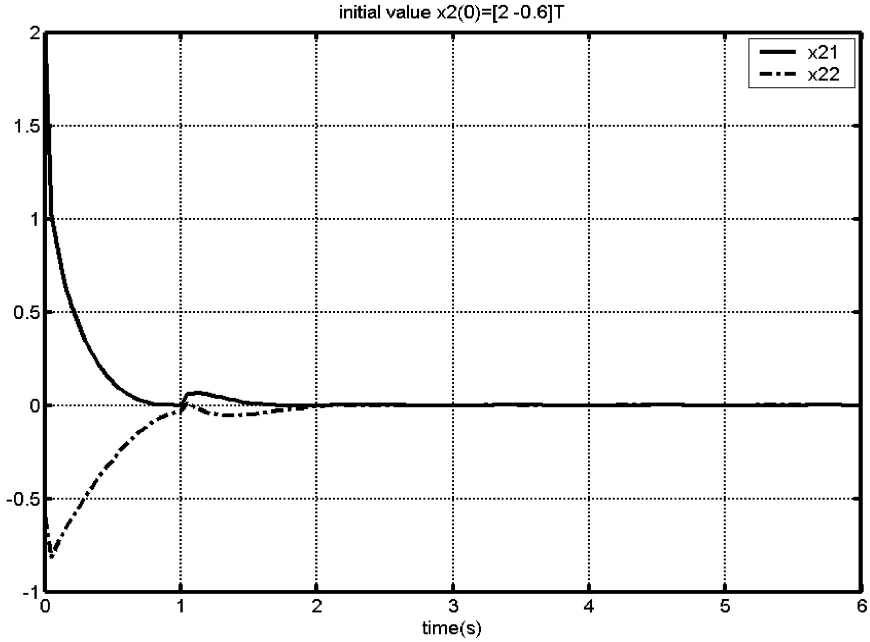

Fig. 5. The state responses of x_{21} and x_{22}.

By Corollary 1, we check inequality (7a), the tolerable bounds are $b_1 = 3.4711,\ b_2 = 2.8625$, and $b_3 = 3.231$. However, if all the time-delays $\tau_{ij}^l(t)$ are the same for all rules (i.e., $\tau_{ij}^l(t) = \tau_{ij}$ for all l and τ_{ij} is constant for $i, j = 1, 2, \ldots, N$). By Corollary 2, we choose the same gain matrices K_i^l as (31) and check the inequality (19a), the tolerable bounds are $b_1 = 4.8162,\ b_2 = 4.0498$, and $b_3 = 4.3761$. Therefore, the design method of Corollary 1 is definitely conservative than that of Corollary 2 in this example. In paper [14], the same controller gain matrices K_i^l are chosen, for all i, l. By Theorem 2 of paper [14], the smaller and more conservative perturbation bounds are obtained $b_1 = 3.2092$, $b_2 = 2.2184$, $b_3 = 3.1424$ ($\varpi_1 = 0.45$, $\varpi_2 = 0.38$, $\varpi_3 = 0.65$).

Figs. 4–6 show the simulation results of three subsystems with time-delay $\tau_{ij}^l(t) = 1$ for all i, j, l, $\psi_i(t) = 0$ for all i, and perturbation bounds are $b_1^l = 4.8162$, $b_2^l = 4.0498$, and $b_3^l = 4.3761$ for all l.

5. Conclusion

Criteria for guaranteeing the stabilizing of the perturbed fuzzy time-delay interconnected systems have been proposed by using the nonlinear decentralized state feedback controllers. Under using our control design methods, we only consider the r_i fuzzy rules for each subsystem then we can find the controller gain K_i^l. This method can eliminate the negative effect of the coupled terms $B_i^l K_i^m$ to relax the stabilized criteria. But meanwhile this method also adds to the other constraint. If $B_i^l = g_l B_i$ and $g_l > 0\ \forall i, l$, then this constraint can be removed. Moreover, if all the time-delays $\tau_{ij}^l(t)$ of each subsystem are the same for all rules (i.e., $\tau_{ij}^l(t) = \tau_{ij}^m(t) = \tau_{ij}$ for all $l \neq m$), we also propose less conservative criteria. These

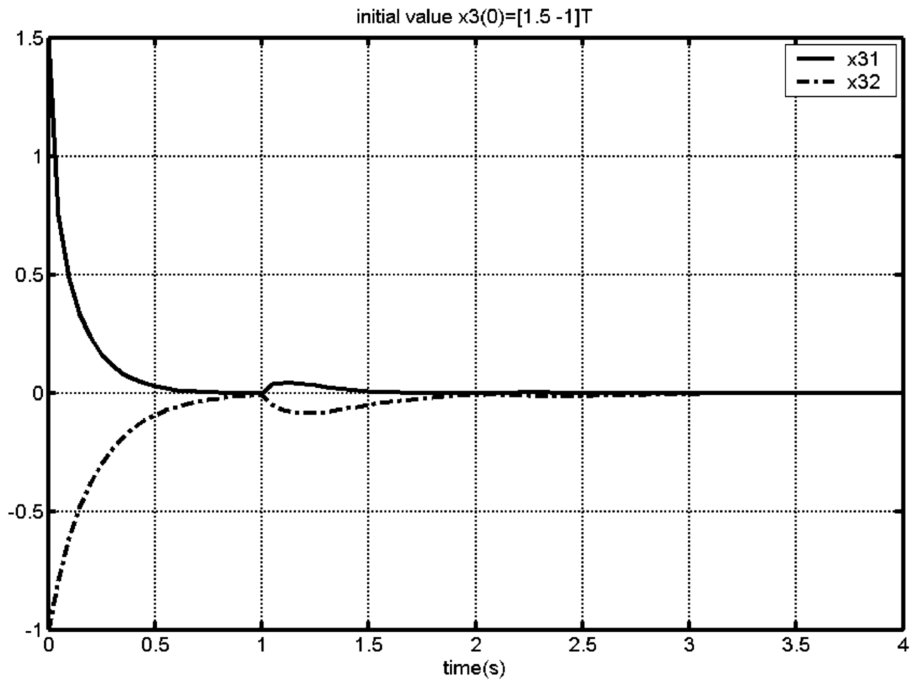

Fig. 6. The state responses of x_{31} and x_{32}.

criteria do not invoke the solution of the Lyapunov equation or the Riccati equation. We also do not need to find a common positive matrix P_i to satisfy any inequality.

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