Reply to “Comment on ‘Spin splitting in modulation-doped Al$_x$Ga$_{1-x}$N/GaN heterostructures’”

Ikai Lo,* J. K. Tsai, W. J. Yao, P. C. Ho, Li-Wei Tu, and T. C. Chang
Department of Physics, Center for Nanoscience and Nanotechnology, National Sun Yat-Sen University, Kaohsiung, Taiwan, Republic of China

S. Elhamri and W. C. Mitchel
Air Force Research Laboratory, MLPS, Wright-Patterson Air Force Base, Dayton, Ohio 45433, USA

K. Y. Hsieh, J. H. Huang, and H. L. Huang
Institute of Materials Science and Engineering, National Sun Yat-Sen University, Kaohsiung, Taiwan, Republic of China

Wen-Chung Tsai
Advanced Epitaxy Technology Inc., 119 Kuangfu N. Road, Hsinchu Industrial park, Taiwan, Republic of China

(Received 10 November 2005; revised manuscript received 14 December 2005; published 25 January 2006)

In this Reply, we reexamine the beating Shubnikov–de Haas oscillations by a nonlinear curve-fitting technique. The results do not support the arguments of Tang et al. [Phys. Rev. B 73, 037301 (2006)], and it is unlikely that the beating Shubnikov–de Haas oscillations we observed in Al$_x$Ga$_{1-x}$N/GaN heterostructures originate from magnetointersubband scattering.

DOI: 10.1103/PhysRevB.73.037302 PACS number(s): 73.21.–b, 71.18.+y, 72.20.Fr, 71.70.Ch

Tang et al. commented on our paper$^1$ that the beating Shubnikov–de Haas (SdH) oscillations we observed in Al$_x$Ga$_{1-x}$N/GaN heterostructures might originate from magnetointersubband scattering (MIS) instead of zero-field spin splitting. To support their argument, they pointed out that (i) a second-subband population with SdH oscillation frequency 16.7 T might exist in sample 3, (ii) the theoretical calculation in wurtzite GaN was still not available, (iii) the phase difference between SdH and MIS oscillations was equal to $\pi$, and (iv) the amplitude of the beating pattern induced by the MIS effect was determined by $A_{\text{MIS}}=\sin n\pi$, where $n=(E_F-E_2)/h\omega_e$.

In order to examine the accurate phases for the individual SdH oscillations, we applied the nonlinear curve-fitting technique to the original data in Fig. 3 of Ref. 1. After the removal of the background noise signal (nonoscillating signal), the oscillatory resistivity $\rho_{\text{osc}}(B)$ was fitted to the superposition of two independent cosine functions,$^2$

$$\rho_{\text{osc}}(B) = \sum_{i=1}^{2} \rho_i \frac{1}{1 + (\mu_i B)^2} \exp \left( -\frac{\pi}{\mu_i B} \right) \frac{1/X_i B}{\sinh(1/X_i B)} \times \cos(2\pi f_i B + \phi_i) ,$$

where $\rho_i$ is a constant proportional to the zero-field resistivity, $\mu_i = \omega_i \tau_i / B$, $\tau_i$ is the quantum lifetime of the carrier, and $\omega_i = eB/m^* i$, $X_i = \hbar e/2\pi k_B T m^*$, and $f_i$ and $\phi_i$ are SdH frequency and phase of the $i$th subband. It is noted that $f_i = n_i \hbar / 2e$ is for the spin-degenerated $i$th subband and $n_i$ is the carrier concentration of that subband, but $f_i = n_i \hbar / e$ is for the spin-splitting subband due to the lift of spin degeneracy.

The fitting results are shown in Fig. 1: the upper black lines are the experimental data and the lower red (gray) lines are the fitting data for each set of different illuminated times. The fitting parameters are shown in Table I.

FIG. 1. (Color online) The nonlinear curve fitting to the SdH data of Fig. 3 in Ref. 1. The upper black lines denote the experimental data and the lower red (gray) lines denote the fitting results for each set.
The two fitting frequencies for zero illumination time (178.8 and 159.3 T) give the carrier concentrations of spin up and spin down, 4.34 and $3.86 \times 10^{12}$ cm$^{-2}$; which is in agreement with the low-temperature carrier concentration determined by Hall measurement, 8.96 T. This is our reply to comment (i) of Tang et al. In addition, none of the phase difference ($\phi_1 - \phi_2$) in Table I is equal to $\pi$. This is our reply to comment (iii) of Tang et al. Based on the theory of magnetointersubband resonant scattering, the beat frequency of the Landau plot is $2.31$ meV. The spin splitting determined by the beat frequency of the Landau plot (Landau levels versus $1/B$), the inset in Fig. 5 of Ref. 1, instead of the $y$-axis intersection of the plot of Das et al. ($\delta$ versus $B$) in Fig. 4 of Ref. 6. The spin splitting determined by the beat frequency of the Landau plot ($f_{\text{beat}}=0.925$ T) is $\Delta \delta=1.16$ meV. The linear fittings of the two plots are shown in Fig. 2, which are used the same data points (sample A of Ref. 6). The results can support our argument. This is our reply to comment (iv) of Tang et al.

A new mechanism ($\Delta_{C1}$-$\Delta_{C3}$ coupling) was recently proposed to describe the large spin splitting in wurtzite GaN, which is originated from the band-folding effect and intrinsic wurtzite structure inversion asymmetry. The band-folding effect generates two conduction bands ($\Delta_{C1}$ and $\Delta_{C3}$), in which the $p$-wave probability has tremendous change when $k_z$ approaches the anticrossing zone. The $\Delta_{C1}$-$\Delta_{C3}$ coupling can produce a spin-splitting energy much larger than traditional Rashba or Dresselhaus effects. This is our reply to comment (ii) of Tang et al.

In conclusion, we have shown that the nonlinear curve-fitting results do not support the arguments of Tang et al. and it is unlikely that the beating Shubnikov–de Haas oscillations observed in Al$_x$Ga$_{1-x}$N/GaN heterostructures originate from magnetointersubband scattering.

This project is supported in part by NSC Core Facilities Laboratory in Kaohsiung-Pintung area, Taiwan (ROC).

### Table I. The parameters of curve fitting results for SdH oscillations in Fig. 1.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$f_1$(T)</th>
<th>$f_2$(T)</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>91</td>
<td>263</td>
<td>1.07</td>
<td>0.31</td>
<td>0.045</td>
<td>0.039</td>
<td>178.8</td>
<td>159.3</td>
<td>82</td>
<td>165</td>
</tr>
<tr>
<td>180</td>
<td>118</td>
<td>172</td>
<td>0.82</td>
<td>0.59</td>
<td>0.058</td>
<td>0.044</td>
<td>179.0</td>
<td>164.3</td>
<td>104</td>
<td>173</td>
</tr>
<tr>
<td>780</td>
<td>126</td>
<td>184</td>
<td>0.80</td>
<td>0.79</td>
<td>0.061</td>
<td>0.041</td>
<td>179.6</td>
<td>164.5</td>
<td>124</td>
<td>157</td>
</tr>
<tr>
<td>2580</td>
<td>129</td>
<td>229</td>
<td>0.94</td>
<td>0.65</td>
<td>0.056</td>
<td>0.040</td>
<td>180.3</td>
<td>164.8</td>
<td>125</td>
<td>146</td>
</tr>
<tr>
<td>7980</td>
<td>142</td>
<td>240</td>
<td>0.80</td>
<td>0.56</td>
<td>0.054</td>
<td>0.042</td>
<td>181.0</td>
<td>164.7</td>
<td>120</td>
<td>148</td>
</tr>
<tr>
<td>13380</td>
<td>149</td>
<td>246</td>
<td>0.79</td>
<td>0.56</td>
<td>0.053</td>
<td>0.041</td>
<td>181.2</td>
<td>164.7</td>
<td>117</td>
<td>147</td>
</tr>
</tbody>
</table>

FIG. 2. (Color online) (a) The spin splitting determined by the $y$-axis intersection of the plot of Das et al. ($\delta$ versus $B$) in Fig. 4 of Ref. 6 is $\Delta \delta=2.31$ meV. (b) The spin splitting determined by the beat frequency of the Landau plot ($f_{\text{beat}}=0.925$ T) is $\Delta \delta=1.16$ meV.


