

Novel SREKF-based recurrent neural predictor for narrowband/FM interference rejection in GPS

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Abstract

The GPS provides accurate positioning and timing information that is useful in various applications. A new adaptive neural predictor for GPS jamming suppression applications is proposed. The effective and computationally efficient square-root extended Kalman filter (SREKF) algorithm is adopted to adjust the synaptic weights in the nonlinear recurrent architecture and thereby estimate the stationary and non-stationary narrowband/FM waveforms. Cholesky factorization is employed in Riccati recursion to improve numerical stability because of the propagation of round-off errors in conventional KF equations. The main characteristics of the proposed SREKF-based canceller are their rapid convergence and favorable tracking performance. Simulation results reveal that its SNR improvement factor exceeds the factors of conventional LMS, RLS, ENA and TLFN filters in single-tone CWI, multi-tone CWI, pulse CWI and FM jamming environments, respectively.

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1. Introduction

GPS satellites broadcast ranging codes and navigation data using the direct sequence spread spectrum (DS-SS). Although DS-SS can tolerate low-power narrowband and wideband obstacles, with its near 43 dB processing gain, it cannot cope with high-power interference. Methods for improving system performance by preprocessing to eliminate intentional or unintentional jamming will be investigated.

According to the GPS signal characteristics, the interference-to-noise ratio (INR) of over 20 dB will prevent the receiver from being able to obtain information on the position. Several approaches in the time- and frequency domains have been employed to mitigate narrowband interference [2,4,11]. The enhanced nonlinear adaptive (ENA) algorithm [2] that is developed to suppress a narrowband

signal outperforms existing linear/nonlinear adaptive filters. The situation becomes more complicated in the presence of wideband interferences [8,9]. Amin proposed a method for rejecting linear FM (chirp-like) interference in GPS communication, based on the time–frequency (TF) representation of the observed signal. The approach consists of evaluating the Wigner–Ville distribution (WVD) of the received signal and then rejecting the undesired signal using the method of subspace projection. The estimation of the instantaneous frequency plays an important role in this FM suppression technique. However, errors in instantaneous frequency may occur under various conditions due to a decrease in interference power, the presence of AM [10] or high levels of cross-terms in TF domain.

The neural network (NN) is a superior nonlinear filtering method for tracking and canceling interference. The pipelined recurrent NN (PRNN) [4] improved SNR more when the statistics and number of CDMA users are

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unknown to the receivers. The fully connected RNNs, which approximate IIR filters, have been demonstrated to outperform the time-lagged feedforward network (TLFN) [6] in non-stationary signal prediction, pattern classification and channel equalization [5,7]. However, conventional gradient-based (GD) approaches such as real-time recurrent learning algorithms use first-order derivative information and converge more slowly than the second-order derivative-based techniques, such as Kalman filter (KF)-trained methods. The KF-based algorithm with both rapid tracking rate and a small prediction error can be used effectively to estimate and remove chirp and narrowband interference in DS-SS applications.

This work presents an SREKF-based recurrent neural predictor (RNP) for GPS anti-jamming applications. The received signal is modeled as the sum of a GPS spread spectrum signal, additive white Gaussian noise and an interfering signal. The adaptive neural filter is employed to predict accurately the narrowband and FM jamming waveforms based on the square-root extended Kalman filter (SREKF) algorithm, which has superior convergence capability. Cholesky factorization is adopted in the recursive KF procedure to solve the Riccati equations; it improves the numerical stability. A detailed computational analysis is conducted and the storage requirements of the SREKF method are examined. The performance of RNP trained with SREKF is evaluated and compared with that of conventional LMS, RLS ENA and TLFN methods in terms of the SNR improvement ratio and the mean-squared prediction error (MSPE) for the interference channels of interest.

2. System description

Fig. 1 presents a simplified block diagram of an anti-jamming GPS system. The observed spread spectrum signal is assumed to have the form [1]:

$$\begin{aligned} r(t) &= [D(t) \oplus CA(t)] \cos(\omega_{L1}t) + \sum_{k=1}^K j_k(t) + n(t) \\ &= S(t) + J(t) + n(t), \end{aligned} \quad (1)$$

where $D(t)$ is the binary data with duration T ($T = 20$ ms). $CA(t)$ represents the binary Gold Code with chip duration T_c ($R_c = 1/T_c = 1.023$ MHz). $\omega_{L1} = 2\pi f_{L1}$ is the $L1$ carrier frequency (1575.42 MHz). $n(t)$ is additive white Gaussian noise with variance σ^2 . The jamming signals $j_k(t)$ may be friendly or intentional. Unintentional sources originate from RF transmitters, which are either onboard an aircraft or at nearby ground RF transmission stations. Intentional signals are always hostile. These three counterparts are assumed to be mutually independent. The jamming source of interest has a bandwidth that is much narrower than the

GPS bandwidth (2.046 MHz). Four forms of interference are investigated.

(1) *Single-tone continuous wave interference (CWI)*:

$$J_{\text{cwi}}(t) = J \cos[(\omega_{L1} + \omega_{\Delta})t + \theta], \quad (2)$$

where J denotes the amplitude and ω_{Δ} is its offset frequency from the central frequency of the spread spectrum signal. θ is the random phase, which is uniformly distributed over the interval $[0, 2\pi)$.

(2) *Multi-tone CWI (MCWI)*:

$$J_{\text{mcwi}}(t) = \sum_{i=1}^I J_i \cos[(\omega_{L1} + \omega_{\Delta_i})t + \theta_i], \quad (3)$$

where J_i , θ_i and ω_{Δ_i} represent the amplitude, random phase and frequency offset, respectively, of the i th interferer, and I is the number of narrowband interferers.

(3) *Pulsed CWI (PCWI)*:

$$J_{\text{pcwi}}(t) = \begin{cases} J \cos(\omega_{L1} + \omega_{\Delta})t, \\ 0, \end{cases} \quad (4)$$

$$(l-1)N_T \leq k < (l-1)N_T + N_1, \quad (4)$$

$$(l-1)N_T + N_1 \leq k < lN_T, \quad (5)$$

where the on interval is $N_1 T_c$ seconds long and the off interval is $(N_T - N_1) T_c$ seconds long. The case in which N_T and N_1 are much greater than unity is considered.

(4) *Linear FM*:

$$J_{\text{FM}}(t) = J \cos \left[(\omega_{L1} + \omega_{\Delta})t + \frac{\alpha_{\Delta}}{2} t^2 \right], \quad (6)$$

where ω_{Δ} and α_{Δ} represent the offset frequency and the frequency rate, respectively.

The received signal is band-pass filtered, amplified and down converted. The received signal is sampled at the chirp rate to simplify the analysis further. The observation at sample k is

$$r(k) = S(k) + j(k) + n(k). \quad (7)$$

Fig. 1 indicates that the proposed interference canceller that consists of a RNP and an adder is utilized to suppress the various jamming signals. This approach exploits the fact that the GPS spreading spectrum signals are difficult to track, whereas a large class of interferences, such as narrowband and FM signals, can be tracked and then canceled. The interference $j(k)$ can be successfully estimated using RNP architecture whose output is the narrowband component of the input signal. Following the subtraction from the received waveform, the output of canceller $z(k)$ is

$$\begin{aligned} z(k) &= r(k) - \hat{j}(k) \\ &= S(k) + n(k) + j(k) - \hat{j}(k) \\ &\approx S(k) + n(k), \end{aligned} \quad (8)$$

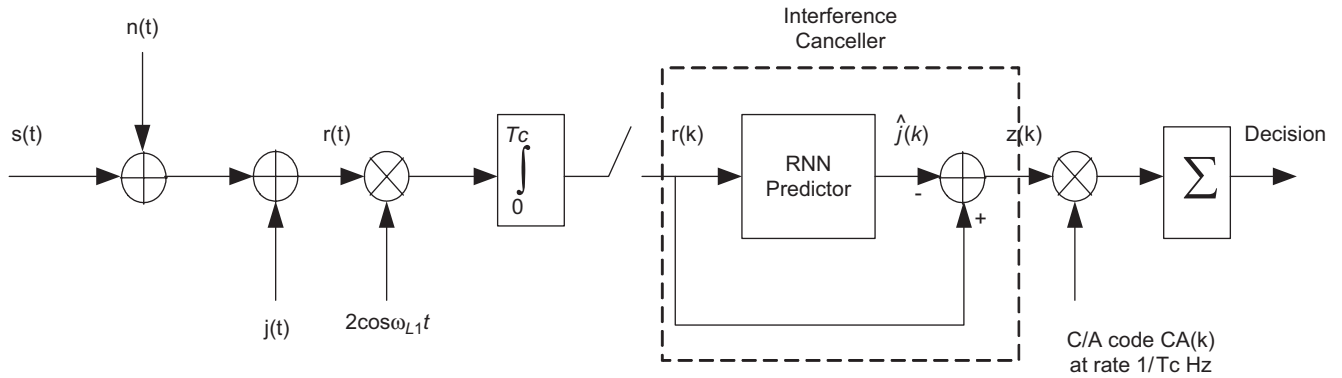


Fig. 1. GPS anti-jamming system.

where $z(k)$ is the broadband component of the received signal. It can be viewed as an almost interference-free signal and is fed into the correlator for despreading GPS.

3. Adaptive SREKF-based RNP

3.1. RNN dynamics

Fig. 2 presents in detail the structure of an RNP. Specifically, the module comprises a fully connected RNP with N hidden neurons, P external input neurons and one output neuron. In each neuron, one-unit delayed version outputs of hidden neurons are assumed to be fed back to the input. Besides the $P + N$ inputs, one bias input whose value is always at +1 is included. The RNN can be described by the following pair of nonlinear state space equations:

$$\begin{aligned} \mathbf{X}(k+1) &= \Phi(\mathbf{W}_a \mathbf{X}(k) + \mathbf{W}_b \mathbf{R}(k)) \\ &= [\phi(\mathbf{W}_1^T \mathbf{U}(k)) \ \dots \ \phi(\mathbf{W}_N^T \mathbf{U}(k))]^T, \end{aligned} \quad (9)$$

$$y(k) = \mathbf{C} \mathbf{X}(k) \quad (10)$$

with

$$\mathbf{R}(k) = [1, r(k), r(k-1), \dots, r(k-P+1)]_{(P+1) \times 1}^T, \quad (11)$$

$$\mathbf{X}(k) = [x_1(k), \dots, x_N(k)]_{N \times 1}^T, \quad (12)$$

$$\mathbf{W} = [\mathbf{W}_a \ \mathbf{W}_b]^T = [\mathbf{W}_1 \ \dots \ \mathbf{W}_j \ \dots \ \mathbf{W}_N]_{L \times N}, \quad (13)$$

$$\mathbf{U}(k) = [\mathbf{X}^T(k) \ \mathbf{R}^T(k)]_{L \times 1}^T, \quad (14)$$

where \mathbf{W}_a represents the synaptic weights of the N neurons in the hidden layer that are connected to the feedback nodes in the input layer, and matrix \mathbf{W}_b represents the synaptic weights of these hidden neurons that are connected to the input nodes. $\mathbf{X}(k)$ is the state vector of an RNN, and $y(k)$ denotes the corresponding output of the system. \mathbf{C} is a $1 \times N$ matrix, which represents the synaptic weights of the output node connected to the hidden neurons. The nonlinear

function $\phi(x) = \tanh(a * x / 2)$ is the sigmoid activation function of a hidden neuron, and a is the gain of a neuron.

3.2. SREKF algorithm

An EKF-based learning algorithm is a second-order, recursive procedure that is particularly effective in training NN architectures. This learning algorithm can be viewed as a means of estimating the state of a nonlinear system, in which the actual response in the output layer is compared with a desired response at each instant. The unforced dynamic and observation equations [5] are

$$\mathbf{w}(k) = \mathbf{w}(k-1), \quad (15)$$

$$\mathbf{y}(k) = \mathbf{h}(\mathbf{w}(k)) + \mathbf{v}(k), \quad (16)$$

where $\mathbf{w}(k)$ is an $L \times 1$ vector that is obtained by rearranging the weight matrix $\mathbf{W}(k)$ into a column vector, and $\mathbf{y}(k)$ is an $N \times 1$ observation vector. $\mathbf{v}(k)$ is an observation noise vector with PDF $\mathbf{v}(k) \sim N(\mathbf{0}, \mathbf{Q})$ (zero-mean white Gaussian noise vector). The nonlinear function $\mathbf{h}(\mathbf{w}(k))$ is assumed to be differentiable and represents a mapping between two Euclidean vector spaces. The Jacobian matrix is defined as the partial derivatives N outputs with respect to the L weights. Hence

$$\mathbf{H}(k) = \frac{\partial \mathbf{h}(\hat{\mathbf{w}}(k))}{\partial \mathbf{w}(k)} = \begin{bmatrix} \frac{\partial h_1}{\partial w_1} & \frac{\partial h_1}{\partial w_2} & \dots & \frac{\partial h_1}{\partial w_L} \\ \frac{\partial h_2}{\partial w_1} & \frac{\partial h_2}{\partial w_2} & \dots & \frac{\partial h_2}{\partial w_L} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h_N}{\partial w_1} & \frac{\partial h_N}{\partial w_2} & \dots & \frac{\partial h_N}{\partial w_L} \end{bmatrix}. \quad (17)$$

The KF-based algorithm is formulated as a sequential minimum mean square error (MMSE) problem, where the weight vector $\mathbf{w}(k)$ is adjusted by minimizing the output power

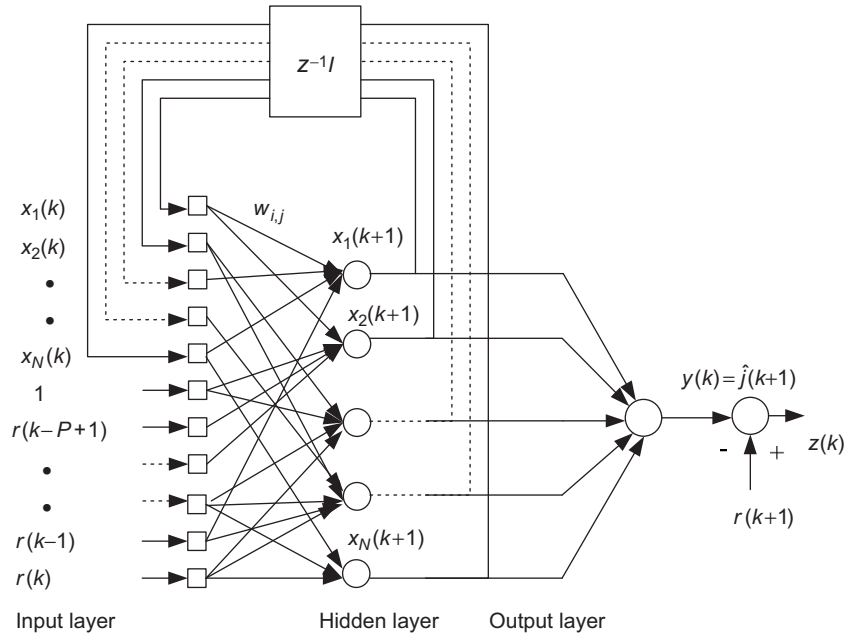


Fig. 2. Block diagram of the recurrent neural predictor (RNP).

using all data so far observed. The cost function is defined as

$$\min_{\mathbf{w}} \sum_{j=1}^k \|z(j)\|^2 = \min_{\mathbf{w}} \sum_{j=1}^k \|\mathbf{y}(j) - \hat{\mathbf{y}}(j)\|^2, \quad (18)$$

where $z(j)$ is the error vector, $\hat{\mathbf{y}}(j)$ is the actual output of network, and $\mathbf{y}(j)$ is the target output. The conventional EKF algorithm is extensionally used to estimate the parameters, but suffers from the serious numerical problem that the error covariance matrix may not remain positive definite. The SREKF, which can effectively mitigate the divergence difficulty, is applied herein. The SREKF solution is derived and formulated by the following recursions:

$$\mathbf{F}(k) = \mathbf{P}^{T/2}(k)\mathbf{H}^T(k), \quad (19)$$

$$\mathbf{G}(k) = [\mathbf{F}^T(k)\mathbf{F}(k) + \mathbf{Q}(k)]^{1/2}, \quad (20)$$

$$\mathbf{K}(k) = \mathbf{P}^{1/2}(k)\mathbf{F}(k)\mathbf{G}(k)^{-T}\mathbf{G}^{-1}(k), \quad (21)$$

$$\hat{\mathbf{w}}(k+1) = \mathbf{w}(k) + \mathbf{K}(k)[\mathbf{y}(k)\mathbf{h}(\hat{\mathbf{w}}(k))], \quad (22)$$

$$\mathbf{P}^{1/2}(k+1) = \mathbf{P}^{1/2}(k) - \mathbf{P}^{1/2}(k)\mathbf{F}(k)\mathbf{G}^{-T}(k) \times [\mathbf{G}(k) + \mathbf{Q}^{1/2}(k)]^{-1}\mathbf{F}^T(k), \quad (23)$$

where $\mathbf{K}(k)$ is the Kalman gain matrix adopted to update the weight matrix and the error covariance matrix. $\hat{\mathbf{w}}(k+1)$ is the estimate of the weight vector $\mathbf{w}(k+1)$ given the data observed up to time k and $\mathbf{P}(k)$ is the error covariance matrix. The covariance square-root matrices can be obtained by Cholesky decomposition: $\mathbf{P}(k) = \mathbf{P}^{1/2}(k)\mathbf{P}^{T/2}(k)$ and $\mathbf{Q}(k) = \mathbf{Q}^{1/2}(k)\mathbf{Q}^{T/2}(k) \times \mathbf{P}^{1/2}(k)$ and $\mathbf{Q}^{1/2}(k)$ are lower triangular square root matrices, and $\mathbf{P}^{T/2}(k)$ and $\mathbf{Q}^{T/2}(k)$ are their transposes.

3.3. Computational complexity

The detailed results of the complexity and memory analyses of the SREKF algorithm are obtained in terms of the number of floating point operations (flops) [6]. The required number of flops in the SREKF method is $\mathbf{O}(4N^2L5 + \frac{5}{2}NL^2 + \frac{1}{2}L^3 + \frac{17}{6}N^3)$, whereas the storage requirement is $\mathbf{O}(L^2 + N^2 + LN)$ for the $\mathbf{F}(k)$, $\mathbf{G}(k)$ and $\mathbf{P}^{1/2}(k)$ matrices. Cholesky factorization is commonly the most computationally burdensome step in numerically solving a positive definite matrix. The computational complexity of an $N \times N$ \mathbf{Q} matrix is $\mathbf{O}(N^3)$ flops.

4. Simulation results

The simulation results of the SREKF-based RNP are obtained to confirm the jamming rejection characteristics. The performance is expressed in terms of SNR improvement and MSPE.

- (1) *SNR improvement*: The metric adopted to verify the steady-state performance is the SNR improvement, which is defined in [2] and given by

$$\text{SNR}_{\text{improvement}} = 10 \log \left[\frac{\mathbb{E}\{|r(k) - S(k)|^2\}}{\mathbb{E}\{|z(k) - S(k)|^2\}} \right] \text{ (dB)}. \quad (24)$$

- (2) *Mean-squared prediction error (MSPE, V_{MSPE})*: The MSPE is used as an index to evaluate the convergence rate of transient responses for various algorithms.

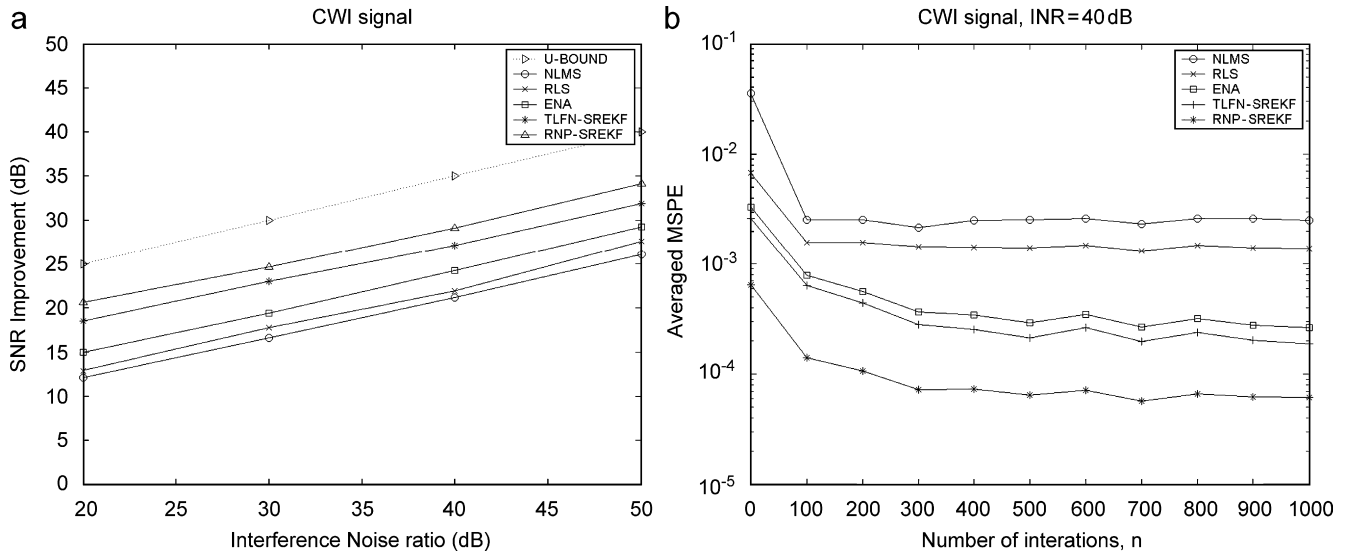


Fig. 3. Single-tone CWI suppression performances of: (a) SNR improvement vs. INR and (b) averaged MSPE vs. the number of iterations.

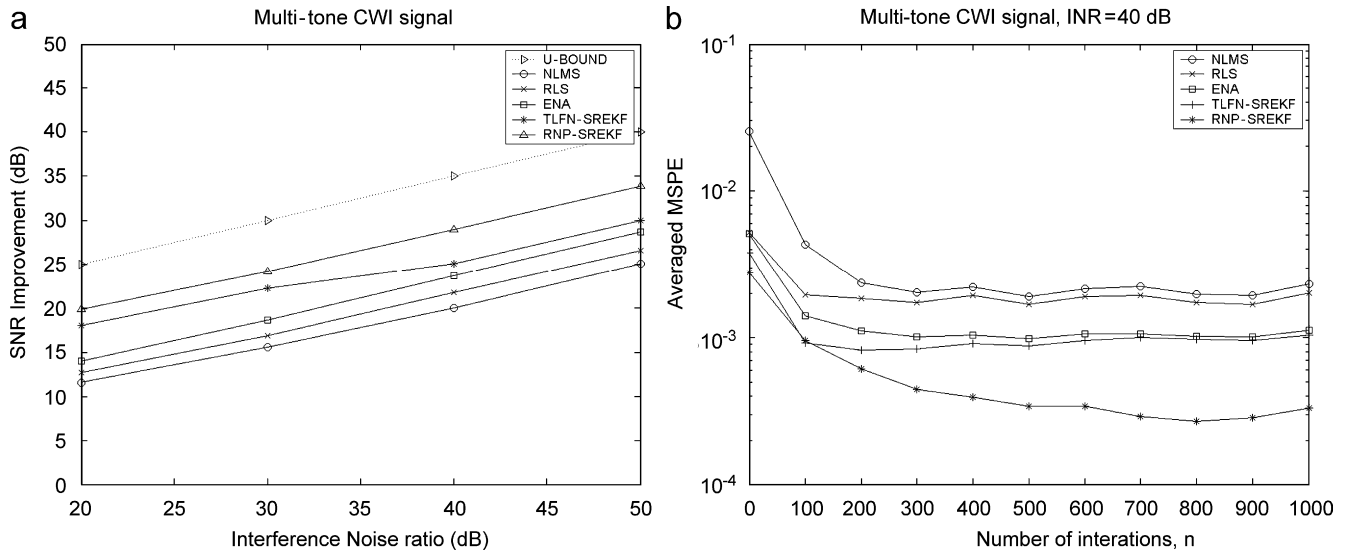


Fig. 4. MCWI suppression performances of: (a) SNR improvement vs. INR and (b) averaged MSPE vs. the number of iterations.

It is defined as

$$V(k) = \frac{1}{\text{SIM}_{\text{num}}} \left(\sum_{i=1}^{\text{SIM}_{\text{num}}} e_i^2(k) \right), \quad (25)$$

$$V_{\text{MSPE}}(n) = \log \left[\frac{1}{100} \left(\sum_{i=100(n-1)+1}^{100n} V(i) \right) \right], \quad (26)$$

where SIM_{num} is the total number of simulations (which is 500 here) and $e_i(k)$ is the predicted error of the k th iteration for the i th run.

In this simulation, the received signal is band-pass filtered, amplified and down converted to IF and then digitized. The

IF is fixed at 1.25 MHz, and a sampling frequency of 5 MHz is selected. $D(k)$ is binomially distributed with a value of ± 1 , and $CA(k)$ is randomly selected with uniform probability from 24 PRN codes of GPS. The variance of thermal noise $n(k)$ is held constant at $\sigma^2 = 0.01$ relative to the signal $S(k)$, the power of which is 1.0. Five techniques are compared; they are LMS, RLS, ENA [2], TLFN-SREKF and RNP-SREKF. The tap numbers of the LMS, RLS and ENA filters are eight. The adaptation constant of LMS is set to 0.01, and the forgetting factors of RLS and ENA are set to 0.9. The TLFN comprises eight input neurons, a multilayer perceptron with 14 hidden neurons, and one linear output neuron. The RNP consists of eight external input neurons, a hidden layer of six recurrent neurons and one linear output neuron. Both TLFN and RNP structures are developed with

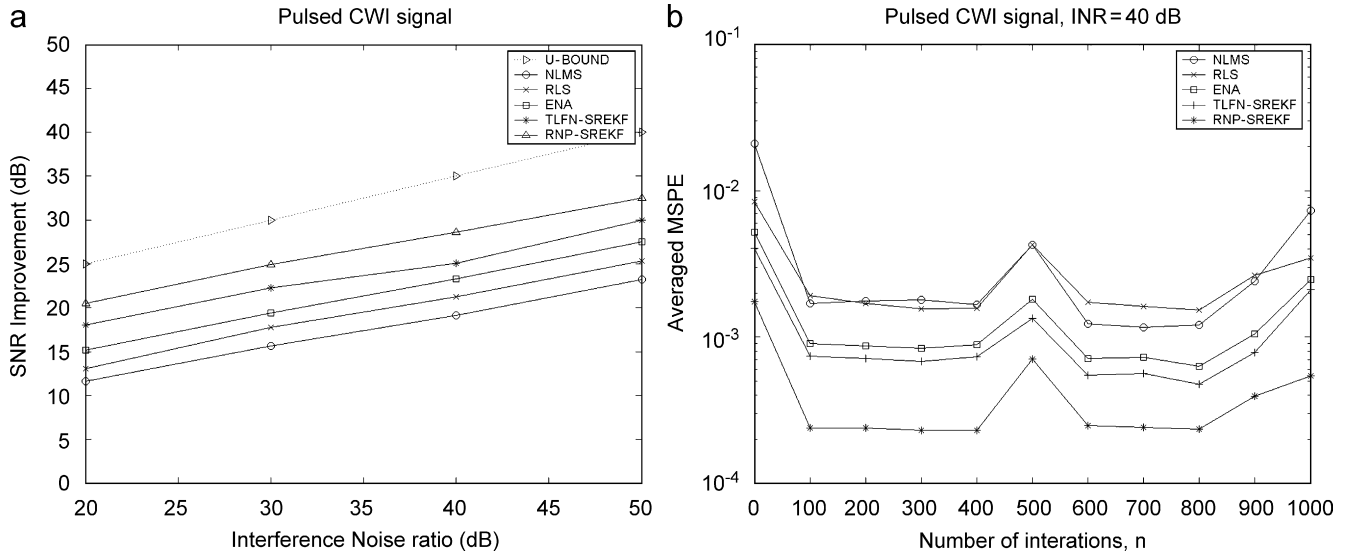


Fig. 5. PCWI suppression performances of (a) SNR improvement vs. INR, (b) averaged MSPE vs. the number of iterations.

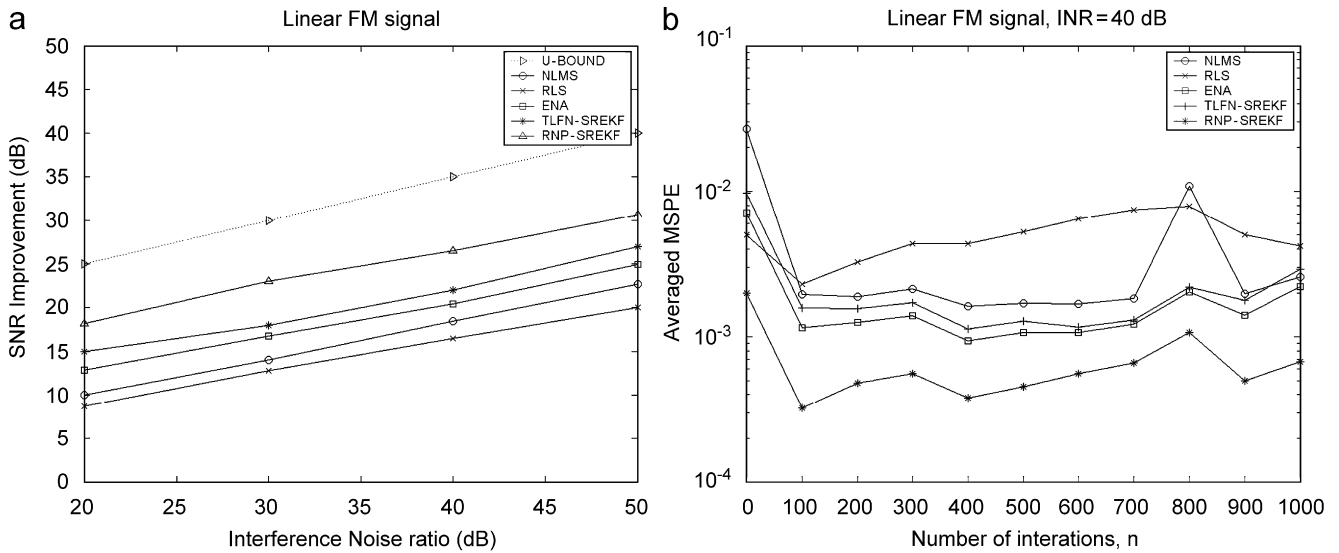


Fig. 6. Linear FM suppression performances of: (a) SNR improvement vs. INR and (b) averaged MSPE vs. the number of iterations.

the same number of adjustable weights to enable fair performance comparison. Also, the initial parameters are selected as $\mathbf{w}(k) = \mathbf{0}$ ($\mathbf{0}$ is a zero vector), $\mathbf{P}(k) = 0.01\mathbf{I}$ (\mathbf{I} is an identity matrix) and $\mathbf{Q} = 0.01$. The simulation results are ensemble averaged over 100 independent runs, and 2000 data points are obtained in each run.

4.1. Stationary jamming signals

Figs. 3 and 4 present the SNR improvements and averaged MSPE for single- and multi-tone CWIs, respectively. The single-tone CWI offset frequency is set to $\omega_A = 1.2$ MHz, and the input INR is varied from 20 to 50 dB. On average,

the RNP-SREKF algorithm yields SNR improvements of 8.15, 7.80, 5.18 and 2.02 dB over the LMS, RLS, ENA and TLFN-SREKF methods, respectively. Fig. 3(b) shows that the RNP-SREKF scheme is also superior in both convergence speed and prediction error. The MSPE can decline significantly to 10⁻⁴ in 250 iterations, while the other methods reach the steady state after 400 iterations and have larger MSPE results. In the second case, a three-tone narrowband jammer is adopted as the severe GPS communication channel. The offset frequencies are set to 0.8, 1.2 and 1.8 MHz. The interfering waveforms can be rejected effectively by the SREKF-training method; the predictor supports faster recursive procedures and learning performance, which are superior to those provided by other methods in the presence of

stationary jamming signals. The RNP–SREKF scheme offers SNR improvements of 8.60, 8.16, 5.42 and 2.61 dB over these four methods, respectively.

4.2. Non-stationary jamming signal

In this PCWI experiment, the frequency offset is set to 0.5 MHz, the on interval to $1000T_c$ and the off interval to $500T_c$. Fig. 5 indicates that the proposed neural predictor offers a faster convergence rate and an SNR improvement over conventional schemes. The SREKF suppression predictor yields SNRs that are 9.22, 9.02, 5.26 and 2.26 dB higher than those of the LMS, RLS, ENA and TLFN–SREKF methods, respectively.

4.3. Linear FM signal

The frequency of the FM signal increases linearly at the beginning of each sweeping interval and is reset at the end of each interval. The frequency rate is set to 10 MHz/s; the sweep bandwidth is set to 1.5 kHz, and the sweeping period includes 750 samples long. Since the SREKF procedure employs an adaptive learning rate (i.e. Kalman gain), in Fig. 6, the convergence is faster and the estimation error is smaller than those obtained by the other methods. The RNP–SREKF yields an SNR that is 3.81, 5.81, 10.03 and 8.24 dB better than those of the LMS, RLS, ENA and TLFN–SREKF methods, respectively.

5. Conclusions

This study presents a new adaptive RNP trained with an SREKF algorithm used to suppress GPS narrowband/FM interference. The proposed canceller, with the powerful recurrent structure, can robustly estimate stationary and non-stationary signals. The SREKF recursions were derived and the corresponding computational analysis and storage requirements presented. On average, the proposed predictor offers SNR improvements of 27.14, 26.67, 26.62 and 24.59 dB in the CWI, MCWI, PCWI and FM environments, respectively. The proposed SREKF-based scheme indeed achieves improved SNR and tracking capability over those of the conventional adaptive filters in various interference circumstances.

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