Losses of the Parallel-Plate Dielectric Resonator

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Abstract – The various loss aspects of the parallel-plate microwave dielectric resonator for the TE$_{01\delta}$ mode is studied from a modified field model. The calculation methods of losses are discussed. The analyses on the calculated results of conductor loss and radiation loss are given. By conducting practical measurements to compare with theoretical calculation results, this modified model has shown dramatic improvement to the unmodified model. Dielectric loss measurement by using this parallel-plate dielectric resonator is also studied.

Keywords – Conductor loss, radiation loss, complex permittivity, dielectric resonator, dielectric measurement.

I. INTRODUCTION

Resonance techniques are widely used for measurements of microwave dielectric properties (complex permittivity, $\varepsilon_r = \varepsilon' + j\varepsilon''$) [1-6], i.e., dielectric constant and dielectric loss (loss tangent, $\tan \delta$). While the dielectric constant does not change much in various microwave frequencies, the dielectric loss always changes under different frequencies. Unfortunately, for most of the resonance techniques, only single frequency point can be measured for each sample. In order to measure the dielectric properties of a different frequency, a new sample with different dimension should be fabricated. The parallel-plate dielectric resonator, Fig. 1, is a very interesting one. By adjusting the distance between the two metal shields, dielectric properties in a quite large range of frequency band can be measured by the parallel-plate dielectric resonator, [6]. By using this technique, we do not need to make several new samples for dielectric measurements on various frequencies, which saves a lot of effort on the preparation of samples.

Various theoretical analyses on the parallel-plate dielectric resonator has been reported in several publications and used for dielectric resonator related designs and dielectric properties (dielectric constant and dielectric loss) measurements in microwave frequencies, [1-5]. The TE$_{015}$ mode is the one always studied. It is easily identified and usually located at the lowest frequency mode. The Itoh and Rudokas model is one of the more interesting theoretical expressions, [1]. This model is modified to give a dramatic increasing on the calculation accuracy of resonant frequency, i.e., accurate measurement of dielectric constant of the dielectric resonator,[2,6].

In addition to the resonant frequency, loss, which related to the attenuation and dispersion of signal propagation, is the other important factor to be studied for the resonator system. Three loss sources may exist in this resonator system -- dielectric loss, conductor loss, and radiation loss. The dielectric loss is from the imaginary part of complex permittivity ($\varepsilon''$) of the dielectric sample. The conductor loss is due to the surface resistance of the two metal shields, which is increased by decreasing the distance between dielectric and shields. Finally, because of the open structure, radiation loss exists in $r$ direction and is increased by increasing the gap between two shields. Regarding to the distance between the two metal shields, a trade-off happens between the conductor loss and radiation loss. Their relation can be expressed as,

$$\frac{1}{Q_T} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_r}$$

(1)

where $Q_d$, $Q_c$, and $Q_r$ are the quality factors due to dielectric loss, conductor loss, and radiation loss, respectively. The $Q_c$ and $Q_r$ are calculated by,
\[ Q_c = \frac{2\omega W_c}{P_c} \]  
\[ Q_r = \frac{2\omega W_r}{P_r} \]  
respectively, where the \( P_c \), \( P_r \), and \( W_c \) are the power lost in conductor, the power lost from the side regions, and the total stored energy. The \( W_c \) and \( P_c \) are computed by,

\[ W_c = \sum_{i=1}^{6} \frac{1}{4} e_i \int_y |E_i|^2 dv \]  
\[ P_c = \sum_{i=2}^{5} \frac{1}{2} \int_{S} R_s |J_{s,i}|^2 ds \]

In the above two equations, the symbol \( i \) indicates the six regions in Fig.1. The \( E \) is the electric field. The \( R_s \) and \( J_s \) are the surface resistance and surface current density of the conducting shields respectively. The calculation of power dissipated by radiation, integration of Poynting’s vector over the surface \( r = b \) is made, where \( b \) is the radius of metal shields.

The \( 1/Q_r \) is the total loss factor of the whole system and can be measured directly. In order to determine the dielectric loss ( \( \tan \delta = \varepsilon''/\varepsilon' = 1/Q_d \) ), it is important to predict the conductor and radiation losses accurately. In this article the conductor loss and radiation loss of the system are calculated by using this modified model. Practical measurements will be conducted to compare the predicted results of both the modified and unmodified models.

II. FIELD MODEL

A cylindrical dielectric rod sample of radius \( a \) and length \( L \) with relative complex permittivity \( \varepsilon_r \) is place between two conducting shield plates with distance \( h \) ( \( h = L_1 + L_2 \) ) as in Fig.1. From Itoh and Rudokas model, the electric fields in each region for the TE\(_{01}\) mode are denoted by the subscripts 1 to 6 and expressed as, \[ E_{\phi_1} = E_o J_1(\alpha L_1) \sin(\alpha L_1) \]  
\[ E_{\phi_2} = E_o \frac{\cos(\beta L + \phi_2)}{\sinh(\alpha L_1)} J_1(\alpha L_1) \sinh(\alpha L_1) \]  
\[ E_{\phi_3} = E_o \frac{\cos(\beta L + \phi_3)}{\sinh(\alpha L_2)} J_1(\alpha L_2) \sinh(\alpha L_2) \]  
\[ E_{\phi_4} = E_o \frac{J_1(k_{ci} a) \cos(\beta L + \phi_1)}{K_1(k_{co} a) \sinh(\alpha L_1)} K_1(k_{co} r) \sinh(\alpha L_1) \]  
\[ E_{\phi_5} = E_o \frac{J_1(k_{ci} a) \cos(\beta L + \phi_2)}{K_1(k_{co} a) \sinh(\alpha L_2)} K_1(k_{co} r) \sinh(\alpha L_2) \]  
\[ E_{\phi_6} = E_o \frac{J_1(k_{ci} a) \cos(\beta L + \phi_3)}{K_1(k_{co} a) \sinh(\alpha L_2)} K_1(k_{co} r) \sinh(\alpha L_2) \]

where

\[ J_0(k_{ci} a) = -k_{ci} a K_0(k_{co} a) \]
\[ k_{ci}^2 = k_{ci}^2 e^{-\beta_1^2} \]
\[ k_{co}^2 = \beta_1^2 - k_{co}^2 \]
\[ k_{co} = \omega \sqrt{\mu_0 \varepsilon_0} \]
\[ \phi_1 + \phi_2 = 0 \]
\[ \phi_{1,2} = \tan^{-1}(\frac{\alpha_{1,2}}{\beta_1} \coth(\alpha_{1,2} L_{1,2}) - \beta_1 L) \]
\[ \alpha_{1,2}^2 = k_{ci}^2 - k_{co}^2 e_{r1,2} \]
\[ k_{co}^2 = \beta_H^2 - k_{co}^2 \]
\[ \beta_H = \frac{\pi}{h} \]

This modification has shown accurate calculation of resonant frequency, \[ 2 \]. The accuracy on the aspects of various losses is yet to be confirmed.

III. EXPERIMENTS

To confirm the validity on loss calculations of the modified model in previous discussions, experiments are conducted to examine the results of computations. The computation results for the unmodified model also will be made to compare with those for the modified one. The experimental setup is shown in Fig. 2. A ceramic specimen
with dielectric constant 23, quality factor due to dielectric loss $Q_d = \frac{81,900}{f\ (GHz)}$, diameter 11.48 mm, and thickness 3.33 mm is adopted for loss measurements. Two brass plates with conductivity $1.41 \times 10^7 \, \text{S/m}$ and diameter 10 cm are used for metal shields.

A network analyzer is used to measure the $S_{21}$ parameter. Two antennas are used to couple in and out the microwave signals. The measured quality factor of the system $Q_T$ is calculated by,

$$Q_T = \frac{f_o}{\Delta f_{3dB} \left| 1 - S_{21} \right|}$$

where, $\Delta f_{3dB}$ and $S_{21}$ are the measured 3dB bandwidth and insertion loss of the TE$_{01\delta}$ mode, respectively, as shown in Fig.3. Fig.4 gives the computed $Q_c$ and $Q_r$ from equations (2) and (3), [7,8]. In addition to the measured total quality factors $Q_T$, by using the results in Fig.4, Fig.5 also gives the $Q_T$ s for the calculated results of modified and unmodified models.

From the experimental results, at first the $Q_T$ values increase with increasing the distance $M$ ($M = L_1 = L_2$)
between the shields and sample. This is expected because of the decreasing conductor loss. After the $Q_T$ values reached a peak, they decrease with increasing the $M$ value because the radiation loss starts dominating the loss mechanism. It can also be found that the computational results by using the unmodified Itoh and Rudokas model cannot correct describe the radiation loss because the decay on the modified Bessel function in equations (6d), (6e) and (6f) is too fast. By using the modified equation (8), the decay of modified Bessel function is much closer to the practical situation. Although the modification method is still not perfect, as most the total losses of experimental results are still higher than the theoretically calculated maximally possible solid curve in Fig.5 which is against the fact, it does give a dramatic improvement to the unmodified model.

IV. SUMMARY

The various aspects of loss for the $TE_{01}$ mode of the parallel-plate dielectric resonator system have been studied. The modified Itoh and Rudokas model of the parallel-plate dielectric resonator was adopted for study. Summation of conductor loss and radiation loss with the added dielectric loss are compared with the practically measured total loss factor. Comparison between the unmodified model and modified model are conducted, this modified model has shown dramatic improvement to the unmodified one. Dielectric constant and loss of dielectric sample can then be measured by this parallel-plate dielectric resonator. With the reasonable prediction of conductor and radiation losses, the dielectric loss of the dielectric resonator sample can then be measured by this parallel-plate dielectric resonator setup by using equation (1), [6]. From equations (7) and (8), the measurement of dielectric constant can be conducted.

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