Amendment of cavity perturbation technique for loss tangent measurement at microwave frequencies

Jyh Sheen
Department of Electronic Engineering, National Formosa University, Huwei, Yunlin 632, Taiwan

(Received 22 April 2007; accepted 20 May 2007; published online 3 July 2007)

Theoretically, the quality factor of a metal resonant cavity will be increased after introducing a lossless dielectric sample. However, this increment of quality factor is not put into the consideration of the calculation of loss tangent measurement for the conventional cavity perturbation method, widely used on the measurements of microwave dielectric properties. Therefore, the conventional resonant perturbation formulas for dielectric loss measurement should be amended. This amendment is introduced in this study and the amended formulas for loss tangent measurement are given. With the amended formulas, no standard sample is required for measurement which simplifies the measuring procedure suggested by the previous publication. In addition, the measuring error on loss tangent on the conventional formula is discussed. Experiments have shown the improvement of measurement accuracy by this modification. © 2007 American Institute of Physics. [DOI: 10.1063/1.2751484]

I. INTRODUCTION

The microwave measurement techniques for dielectric properties measurements can be divided into two groups: (i) resonance techniques and (ii) transmission techniques. The resonance techniques do not have the swept frequency capability. Only one or certain frequency points can be measured. All three space dimensions are considered for the computation of dielectric properties. Unlike the resonance techniques, the transmission techniques usually have the swept frequency ability for the measured frequency range. The transmission and/or reflection signals are always tested to calculate the dielectric properties of the specimen. Only one dimension (direction of signal propagation) is mainly considered.1,2 However, dielectric properties measurements by the resonance techniques have higher accuracy than measurements by the transmission techniques especially for the dielectric loss.3 Therefore, resonance techniques are still widely used. The resonance techniques can be further divided into two categories: The first is that the resonance is basically supported by the dielectric sample itself.3–10 The sample acts as a dielectric resonator. Metal shields with different geometries are always introduced to prevent radiation loss. This type is called dielectric resonance technique. The second type is that the resonance is supported by the metal walls of the metal cavity.11–20 The presence of samples in the cavity causes only a “perturbation” on the field distributions in the metal cavity. The second type is called the perturbation technique and is studied in this article.

The fundamental concept of the perturbation technique is that the presence of a small piece of the dielectric sample in the resonant cavity will cause a shift of resonant frequency and a decrease of the quality factor of the cavity. The “small” means that the volume of the sample is much smaller than the volume of the cavity. It is also assumed the presence of the specimen to the change in the overall geometrical configurations of the electromagnetics must be very small. The complex permittivity of the specimen can then be calculated from the changes of resonant frequency and quality factor of the metal cavity. The rectangular or cylindrical cavities as shown in Figs. 1 and 2 are usually adopted since the equations for the fields are easier to derive for simple geometries. The sample is always put in the position of maximum electric field for testing, where the equations for the calculations of dielectric constant and loss can be derived more easily than by putting the specimen in the nonmaximum position.

The main advantages of the cavity perturbation technique are:

(1) Unlike the dielectric resonant techniques measurements, there is really no severe tolerance limit on the shape and dimension of measured specimen;3–10

(2) Requirement for specimen size is very small. Usually, a small piece of rod, sheet, or bar shaped sample is adopted for measurements. The standard cross section of the sample is about 1 mm², therefore the sample is easily prepared. Some microwave techniques require large

FIG. 1. TE cavity perturbation technique. The a, b, and d are the width, height, and length of the cavity, respectively.
II. CONVENTIONAL THEORETICAL ANALYSIS

A. Rectangular cavity

For the rectangular cavity, the TE\textsubscript{10N} modes (N is an integer) are widely used for the complex permittivity measurements.\textsuperscript{11,12} The sample is placed in the position of maximum electric field. Usually there are N positions in the cavity with maximum electric field for the TE\textsubscript{10N} modes.

Odd modes (N: odd) are more convenient used because the geometrical center is always one of the maximum electric field positions despite the fact that the maximum field positions change with changing N values. For even modes, the maximum electric field positions are more difficult to locate because the center position is no longer one of the positions of maximum electric field. The specimen position will have to be moved with different TE\textsubscript{10N} modes.

The complex permittivity is calculated from the changes of resonant frequency and quality factor. The following two equations give the theoretical calculations of resonant frequency \( f_c \) and quality factor \( Q \) for TE\textsubscript{10N} modes of the rectangular cavity:

\[
f_c = \frac{c}{2} \left[ \left( \frac{1}{a} \right)^2 + \left( \frac{N}{d} \right)^2 \right]^{1/2},
\]

\[
Q = \frac{(kad)^3 b \eta}{2\pi^2 R_s (2N^2 a^3 b + 2bd^3 + N^2 a^3 d + ad^3)}.
\]

The earlier two equations can be found in a lot of electromagnetic wave literature, where \( c \) is the speed of the light in free space, \( \eta \) is the intrinsic impedance, \( R_s \) is the surface resistance of the cavity, and \( a, b, \) and \( d \) are the dimensions as shown in Fig. 1. By measuring the resonant frequency \( f_c \), the half power bandwidth \( \Delta f_{3/4} \), insertion loss \( S_{21} \), and the measured \( Q \) value of the cavity is calculated by the following equation:\textsuperscript{21}

\[
Q = \frac{f_c}{\Delta f_{3/4} |1 - S_{21}|}.
\]

The basic equation of cavity perturbation theory can be derived\textsuperscript{22}

\[
2\left( \frac{f_c - f_s}{f_s} \right) - \int \left( \frac{1}{Q_c} - \frac{1}{Q_s} \right) \frac{V_c E_1 E_2 dV}{V_c |E|^2 dV} = (\varepsilon_r - 1) \frac{\int V_c E_1 E_2 dV}{\int V_c |E|^2 dV},
\]

where \( f_c \) and \( f_s \) are the resonant frequencies, \( Q_c \) and \( Q_s \) are the measured quality factors of the cavity without and with a lossy sample inside the cavity, respectively; \( V_c \) and \( V_s \) are the volumes of cavity and the sample, and \( E_1 \) and \( E_2 \) are the electric fields in the cavity and sample, respectively. The \( \varepsilon_r = \varepsilon' - j\varepsilon'' \) is the relative complex permittivity. From Eq. (3), the changes of \( f_c \) and \( Q \) of the cavity can be expressed as

\[
2\left( \frac{f_c - f_s}{f_s} \right) = (\varepsilon' - 1) \frac{\int V_c E_1 E_2 dV}{\int V_c |E|^2 dV},
\]

\[
\frac{1}{Q_c} - \frac{1}{Q_s} = \varepsilon'' \frac{\int V_c E_1 E_2 dV}{\int V_c |E|^2 dV}.
\]

Equation (4b) requires modification. From Eq. (4b), for a lossless material \((\varepsilon''=0)\), the \( Q_s \) will not change \((Q_s=Q_c)\). Actually, the \( Q_s \) will increase if the specimen is lossless. The theoretical amendment will be given later. As mentioned before, the basic assumption for the cavity perturbation technique is that the change of electromagnetic field upon the introduction of the sample must be small. From the assumption, by inserting the sample symmetrically in a region of maximum electric field, the real and imaginary parts of the
relative complex permittivity are calculated by\(^{11,12}\)
\[
e' = \frac{V_c (f_c - f_s)}{2 V s f_s} + 1, \quad (5a)
\]
\[
e'' = \frac{V_c}{4 V s} \left( \frac{1}{Q_s} - \frac{1}{Q_c} \right). \quad (5b)
\]

The dielectric loss \(\tan \delta\) and quality factor \(Q_d\) due to the dielectric loss of the measured material can be expressed as
\[
1 = \tan \delta = \frac{e''}{e'}. \quad (6)
\]

B. Cylindrical cavity

For the cylindrical cavities, the TM\(_{010}\) mode of the cylindrical cavity is usually used. The sample was inserted along its symmetric axis in the position of maximum electric field strength for easy measurement and calculation as shown in Fig. 2. The theoretical resonant frequency and quality factor of the TM\(_{010}\) mode of the cavity can be calculated as\(^{13}\)
\[
f_c = \frac{11.5}{a \text{(cm)}} \text{ GHz}, \quad (7a)
\]
\[
Q = \frac{453}{R_s \left( \frac{a}{d} + 1 \right)}, \quad (7b)
\]
where \(a\) and \(d\) are the inside radius and height of the cavity, respectively. The resonant frequency depends only on the radius of the cavity because the field varies only in the \(r\) direction. Equation (3) used by the rectangular cavity can also be simplified to calculate the dielectric constant and the loss value for the cylindrical cavity\(^{13}\)
\[
e' = 0.539 \frac{V_c (f_c - f_s)}{V s f_s} + 1, \quad (8a)
\]
\[
e'' = 0.269 \frac{V_c}{V s} \left( \frac{1}{Q_s} - \frac{1}{Q_c} \right). \quad (8b)
\]

One disadvantage of the cylindrical cavity method is that only a single frequency point can be measured for one cavity, while the rectangular cavity has several TE\(_{10n}\) modes for measurements.

III. THEORETICAL AMENDMENT

For the computations of the imaginary part of the dielectric constant in Eqs. (5b) and (8b), it is assumed that the quality factor \(Q_c\) does not change before and after the insertion of a lossless dielectric sample. This means the stored energy in the empty cavity equals that in the cavity with the lossless sample. However, the \(Q_c\) is calculated with the following equations:
\[
Q_c = \frac{2 \omega W_c}{P_c}, \quad (9a)
\]

\[\]

\[
P_c = \frac{1}{2} R_s \int_s |E_s|^2 dS, \quad (9b)
\]
\[
W_c = \frac{1}{4} e' \int_v |E|^2 dV, \quad (9c)
\]

where the \(P_c\) and \(W_c\) are the power lost in conductor and the total stored energy. From Eq. (9c), the stored energy may change with both the dielectric constant and electric field intensity. The interface condition in electromagnetic theory tells us that the tangential component of electric field is continuous between the dielectric and free space. Therefore, we can assume a uniform distribution of electric field in the cavity. The stored energy is then proportional to the real part of the complex permittivity. Therefore, the quality factor of a metal resonant cavity will be increased after introducing a lossless dielectric sample. This increment of quality factor should be put into the consideration of calculation of loss tangent measurement. The conventional resonant perturbation formulas in Eqs. (5a), (5b), (8a), and (8b) for dielectric loss measurement were then amended.

IV. DERIVATIONS OF AMENDED FORMULAS

In Refs. 14 and 15 the earlier amendment’s concept was introduced for measurements of dielectric properties. There, before measuring the desired specimen, with the basis of Eq. (4a) and (4b), one medium-low standard dielectric sample with known permittivity was required to calibrate the cavity to have values of \(\int V E dV / \int V |E|^2 dV\) for Eq. (4a) and (4b). In addition, a set of extremely low-loss standard dielectric samples are suggested to have accurate \(Q_c\) in both Eqs. (5a), (5b), (8a), and (8b). The procedures for measurements are then not very convenient.

A direct and simple modification to the Eqs. (5), (5b), (8a), and (8b) will be given in the following. There is no standard sample that will be required for dielectric properties measurements by using the modified equations. Simply measure the parameters in the modified equations, the dielectric properties of the specimen can be calculated directly. Therefore, the measurement procedures can be dramatically simplified.

To execute the amendment on \(Q_c\), we assume that the contribution of the quality factor of the cavity is based on the volume percentages of the sample and free space inside the cavity. In addition, the quality factor is proportional to the dielectric constant \(e'\). Before inserting the sample in the cavity, the cavity’s quality factor is \(Q_c\). After inserting a lossless sample, the quality factor becomes \(Q'_c\). The calculation of \(Q'_c\) is then given by
\[
Q'_c = Q_{\text{free space}} + Q_{\text{lossless sample}}
\]
\[
= Q_c \times \text{volume percentage of free space} + e' \times Q_c \times \text{volume percentage of sample}
\]
\[
= Q_c \frac{V_c - V_s}{V_c} + Q_c \frac{V_s}{V_c} e' = Q_c \left[ 1 + (e' - 1) \frac{V_s}{V_c} \right]. \quad (10)
\]

This amendment formula is used to replace the \(Q_c\) in Eqs.
(5b) and (8b) for loss tangent measurement. For rectangular cavity, Eq. (5a) and (5b) is modified to
\[
e' = \frac{V_c(f_c - f_s)}{2V_s f_s} + 1, \tag{11a}
\]
\[
e'' = \frac{V_c}{4V_s} \left( \frac{1}{Q_s} - \frac{1}{Q_c'} \right). \tag{11b}
\]
For cylindrical cavity, Eq. (8a) and (8b) is modified to
\[
e' = 0.539 \frac{V_c(f_c - f_s)}{V_s f_s} + 1, \tag{12a}
\]
\[
e'' = 0.269 \frac{V_c}{V_s} \left( \frac{1}{Q_s} - \frac{1}{Q_c'} \right). \tag{12b}
\]
Without the modification to the earlier equations, one can expect that the measured loss tangent can go to zero or negative. From Eq. (10), for \((e' - 1)V_c/V_c' \ll 1\), we can obtain
\[
\frac{1}{Q_c} - \frac{1}{Q_c'} \approx \left[ 4e'' - (e' - 1) \frac{1}{Q_c} \right] \frac{V_s}{V_c}. \tag{13}
\]
Therefore, for \(4e'' < (e' - 1)/Q_c\), if the amendment was not given, a negative loss tangent will be measured. Of course, it is physically impossible.

In order to understand how much of the measurement accuracy can be improved by this theoretical amendment, the measurement error without amendment is discussed in the following.

For the rectangular cavity, from Eq. (5b) or (11b), the measurement error of the conventional perturbation method on the imaginary part of complex permittivity is
\[
d\varepsilon'' = \frac{V_c}{4V_s Q_c^2} dQ_c = \frac{1}{4} (e' - 1) \frac{1}{Q_c} \approx \frac{1}{4} e' \quad \text{for} \quad e' \gg 1,
\]

where, from Eq. (10), \(dQ_c = (e' - 1)(V_s/V_c)Q_c\). We have, for \(e' \gg 1\),
\[
d\tan \delta = \frac{1}{4} \frac{1}{Q_c}. \tag{15}
\]

The measurement error on loss tangent for the rectangular cavity is then
\[
\Delta \tan \delta = \frac{1}{4} \frac{1}{Q_c} \tan \delta = \frac{1}{4} \frac{Q_d}{Q_c}. \tag{16}
\]

For the cylindrical cavity, the measurement error is expressed as
\[
\Delta \tan \delta = 0.269 \frac{1}{Q_c} \frac{1}{Q_c} = 0.269 \frac{Q_d}{Q_c}. \tag{17}
\]

It can be seen that the measured error is increased by increasing the quality factor of measured sample and by decreasing the quality factor of resonant cavity, respectively. For a typical quality factor for empty cavity of 2500–5000, the measurement error for a material with loss tangent \(10^{-4}\) can be as high as 50%–100%. The relationship between the measurement error caused by the traditional formula and the loss tangent of measured material is shown in Fig. 3. From Fig. 3, for high loss materials, the increment of \(Q_c\) due to the insertion of a lossless sample is much smaller than the reduction of the \(Q_c\) due to the sample’s dielectric loss. Therefore, the measurement errors are small. However, for low loss materials, the increment of \(Q_c\)
errors increase with decreasing the dielectric loss.

For comparison. Since the Eq. (5a) and (5b) were derived for high dielectric constant materials. The measured quality factor $Q_c$ of empty cavity is 4110 with resonance mode of 10.16 GHz (TE$_{107}$).

The measurement results are listed in Table I. For loss tangent measurements, both the measurement errors by Eq. (5a) and (5b) and the estimated errors by Eq. (16) are listed for comparison. Since the Eq. (16) was derived for $\varepsilon' \gg 1$, the differences on the last two columns in Table I are larger for low dielectric constant materials.

### V. EXPERIMENTS

To examine the improvement of using this amended perturbation technique, samples were measured by this cavity perturbation technique under a rectangular cavity. An X-band (8.2–12.4 GHz) rectangular cavity fabricated by standard WR-90 copper waveguide with inside cross section 2.29 cm × 1.02 cm and length 13.5 cm was adopted for measurements. The measured quality factor $Q_c$ of empty cavity is 4110 with resonance mode of 10.16 GHz (TE$_{107}$).

The measurement results are listed in Table I. For loss tangent measurements, both the measurement errors by Eq. (5a) and (5b) and the estimated errors by Eq. (16) are listed for comparison. Since the Eq. (16) was derived for $\varepsilon' \gg 1$, the differences on the last two columns in Table I are larger for low dielectric constant materials.

### VI. DISCUSSIONS AND CONCLUSIONS

The difference between the previous work on cavity perturbation technique with this article is this: some of the previous published work requires standard samples for calibration before measuring the desired specimen, the measurement procedure is much more complicated and other work exists higher measuring error on loss tangent by using the conventional formulas as shown in Fig. 3; while this article gives the direct calculation formulas to simplify the procedure of permittivity measurements and amends the error on loss tangent measurement by the conventional formulas.

In conclusion, theoretical modification on the cavity perturbation technique for loss tangent measurement has been given in this article. The modified formulas for loss tangent measurement on both the rectangular cavity and cylindrical cavity are derived. This modification is based on the increment of cavity quality factor after inserting a lossless dielectric sample. The error on dielectric loss measurement from the unmodified formulas was discussed. It was found the measured error is increased by increasing the quality factor of measured sample and by decreasing the quality factor of resonant cavity, respectively. Simply conducting the measurements of the parameters in Eqs. (11a), (11b), (12a), and (12b), the dielectric properties of the specimen can be easily calculated. The errors on loss tangent measurements estimated by Eq. (16) agree with the experimental values for high dielectric constant materials.

### ACKNOWLEDGMENT

This work was partially supported by the National Science Council of Taiwan under Contract No. NSC 92-2213-E-161-003.

---