Losses of the parallel-plate dielectric resonator

J. Sheen

Department of Electronic Engineering, National Formosa University, Hu-Wei, Yun-Lin 632, Taiwan, Republic of China
E-mail: jsheen@nfu.edu.tw

Abstract: The various loss aspects of the parallel-plate microwave dielectric resonator for the TE_{01\delta} mode is studied from a modified field model. The main merit of this field model is that it provides very simple electromagnetic field expressions of this field mode. Various methods of calculation of losses are discussed. The analyses of the calculated results of conductor and radiation losses are given. The relationship of the loss with the dimensions of the device is introduced. By conducting practical measurements to compare with theoretical calculation results, this modified model has shown dramatic improvement over the unmodified model. The dielectric loss measurement by using this parallel-plate dielectric resonator is also studied.

1 Introduction

Resonance techniques are widely used for measurements of microwave dielectric properties (complex permittivity, \( \varepsilon_r = \varepsilon - j\varepsilon'' \)) [1–9], i.e. dielectric constant and dielectric loss (loss tangent, \( \tan \delta = \varepsilon''/\varepsilon' \)). Although the dielectric constant does not change much in various microwave frequencies, the dielectric loss always changes under different frequencies. Unfortunately, for most of the resonance techniques, only a single frequency point can be measured for each sample [1]. To measure the dielectric properties of a different frequency, a new sample with different dimensions should be fabricated. The parallel-plate dielectric resonator, Fig. 1, is very interesting. By adjusting the distance between the two metal shields, dielectric properties in quite a large range frequency band can be measured by the parallel-plate dielectric resonator [2]. By using this technique, one does not need to make several new samples for dielectric measurements on various frequencies, which saves a lot of effort on the preparation of samples.

Various theoretical analyses on the parallel-plate dielectric resonator in Fig. 1 have been reported in several publications [3–9] and adopted for dielectric resonator related designs and dielectric property (dielectric constant and dielectric loss) measurements in microwave frequencies. The resonant modes and influence of conducting shields on Q-factors have been studied by Kobayashi et al. [3, 4]. Lui and Wu [5] present an efficient volume integral equation approach for the characterisation of lossy dielectric materials. Prokopenko et al. [6–8] have studied the parallel-plate dielectric resonator of the anisotropic disk. The surface resistance measurement of high Tc superconductor films using a sapphire resonator has also been studied [10]. Dielectric resonators with closed shield structure have been studied by Krupka et al. [11–14] and accurate measurements of microwave complex permittivity are given. Theoretical analyses on a similar structure have also been reported elsewhere [15–22].

Although very accurate theoretical analyses have already been given by previous publications, their analyses are more complicated than the Itoh and Rudokas model [23]. The Itoh and Rudokas model is an attractive theoretical model because its electromagnetic field expressions are very simple. This model is a single mode expression, the TE_{01\delta} mode. This mode is easily identified and usually located at the lowest frequency mode. The accuracy of predicting the resonant frequency by this model is improved by a modification to this model. This modification gives a dramatic enhancement on the calculation accuracy of the resonant frequency [24], i.e. accurate measurement of the dielectric constant of the dielectric resonator [2].
Figure 1 Parallel-plate dielectric resonator

In addition to the resonant frequency, loss, which is related to attenuation and dispersion of signal propagation and plays a key role for loss tangent measurement, is the other important factor to be studied for the resonator system. It is going to be a very interesting topic whether this simple field model can accurately predict losses or not.

The accuracy of predicting losses by this model has been studied by very limited theoretical analyses and experimental results of a single ceramic specimen [25]. Losses of the parallel-plate dielectric resonator are then further investigated by this modified model. Calculations of the conductor and radiation losses of the system by using this modified model will be studied. Conductor loss calculations by using two different methods will be compared. Relationship of losses with various dimensions of the sample, air gap and shield will be discussed. In addition, practical measurements with three ceramic specimens will be conducted to compare the predicted results by both the modified and unmodified models. Dielectric loss measurement through this parallel plate dielectric resonator setup is also studied.

2 Field model

A cylindrical dielectric rod sample of diameter $D$ and length $L$ with relative complex permittivity $\varepsilon_r = \varepsilon' - j\varepsilon''$, is placed between two conducting shield plates with distance $h$ as shown in Fig. 1. From the Itoh and Rudokas model, the electric fields in each region for the $TE_{010}$ mode are denoted by the subscripts one to six and expressed as [23, 24]

$$E_{\phi_1} = E_o j_1(k_{r1}r) \cos (\beta_1 z - \phi_1) \quad (1a)$$

$$E_{\phi_2} = E_o \frac{\cos (\beta_1(L/2) + \phi_1)}{\sin (\alpha_1 L_1)} j_1(k_{r2}r) \times \sinh \alpha_1 \left( z + L_1 + \frac{L}{2} \right) \quad (1b)$$

$$E_{\phi_3} = E_o \frac{\cos (\beta_2(L/2) + \phi_2)}{\sin (\alpha_2 L_2)} j_1(k_{r3}r) \times \sinh \alpha_2 \left( L_2 + \frac{L}{2} - z \right) \quad (1c)$$

$$E_{\phi_4} = \frac{E_o j_1(k_{r4}r)}{K_1(k_{r4}a)} \frac{\cos (\beta_2(L/2) + \phi_2)}{\sinh (\alpha_2 L_1)} K_1(k_{r4}r) \times \sinh \alpha_2 \left( z + L_1 + \frac{L}{2} \right) \quad (1d)$$

$$E_{\phi_5} = \frac{E_o j_1(k_{r5}r)}{K_1(k_{r5}a)} \frac{\cos (\beta_2(L/2) + \phi_2)}{\sinh (\alpha_2 L_2)} K_1(k_{r5}r) \times \sinh \alpha_2 \left( L_2 + \frac{L}{2} - z \right) \quad (1e)$$

$$E_{\phi_6} = \frac{E_o j_1(k_{r6}r)}{K_1(k_{r6}a)} K_1(k_{r6}r) \cos (\beta_2 - \phi_1) \quad (1f)$$

where

$$J_0(k_{r1}a) - \frac{k_{r2}a K_0(k_{r2}a)}{k_{r4}a K_1(k_{r4}a)} = 0 \quad (2a)$$

$$k_{r1}^2 = k_{r2}^2 \varepsilon' - \beta_1^2 \quad (2b)$$

$$k_{r2}^2 = \beta_1^2 - k_{r1}^2 \quad (2c)$$

$$k_o = \omega \sqrt{\mu \varepsilon_o} \quad (2d)$$

$$\phi_1 + \phi_2 = 0 \quad (2e)$$

$$\phi_{1,2} = \tan^{-1} \left( \frac{\alpha_{1,2}^2}{\beta_{1,2}} \coth \alpha_{1,2} L_{1,2} \right) - \beta_{1,2} L_{2,1} \quad (2f)$$

$$\alpha_{1,2}^2 = k_{r1}^2 - k_{r2}^2 \varepsilon_{1,2} \quad (2g)$$

and $J$ and $K$ are Bessel function and modified Bessel function, respectively. The magnetic field components can easily be derived from the above equations.

On the boundaries of regions four and six and regions five and six, the field tangential components are not continuous for this simplified model. It will be shown that this theoretical defect will not influence the effective calculation of losses. With the modified model, (2c) is replaced by [24]

$$k_{r4}^2 = \beta_{11}^2 - k_{o}^2 \quad (3a)$$

$$\beta_{11} = \frac{\pi}{h} \quad (3b)$$

This modification has shown accurate calculation of the resonant frequency [24]. The relationship among the resonant frequency, dielectric constant $\varepsilon'$ and air gap of the sample and shields with $M = L_1 = L_2$ is shown
in Fig. 2. Various loss aspects of the system is then studied by this modified model.

3 Computations of conductor and radiation losses

Three loss sources may exist in this resonator system – dielectric loss, conductor loss and radiation loss. Their relationship can be expressed as

\[ \frac{1}{Q_T} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_r} \]  

(4)

where \( Q_d \) (=1/\( \tan \delta \)), \( Q_c \) and \( Q_r \) are the quality factors because of dielectric loss, conductor loss and radiation loss, respectively. The 1/\( Q_T \) is the total loss factor of the whole system. For \( h \geq \lambda_s/2 \) (\( \lambda_s \) is the free space wavelength corresponding to the resonant frequency), the distance between the two shields is over the cutoff condition and \( k_{c1} \) becomes imaginary, as can be seen from (3a) and (3b). The modified Bessel function in (1d), (1e) and (1f) will be replaced by the Hankel function and the radiation loss is dramatically increased [3]. Then \( Q_c \) and \( Q_r \) are calculated by [26]

\[ Q_c = \frac{2\omega W_c}{P_c} \]  

(5a)

\[ Q_r = \frac{2\omega W_r}{P_r} \]  

(5b)

respectively, where the \( P_c \), \( P_r \) and \( W_c \) are the power lost in the conductor, the power lost from the side regions and the total stored electric energy. \( P_c \) and \( W_r \) are computed by

\[ P_c = \sum_{j=2}^{5} \frac{1}{2} R_s \int |j_{s,i}|^2 dS \]  

(6a)

\[ W_r = \sum_{j=1}^{6} \frac{1}{4} e' \int |E_i|^2 dV \]  

(6b)

where the \( R_s \) and \( J_s \) are the surface resistance and surface current density of the metal shields.

There is another method for the calculation of conductor loss. By following a perturbation method given by Kajfez [27]: if the metal shields are moved a very small distance \( \Delta M \) (\( M = L_1 = L_2 \)), the resonant frequency \( f_o \) will have a very small shift \( \Delta f_o \) and the quality factor from the conductor loss can then be calculated by [4, 27]

\[ Q_c = -\frac{f_o}{\Delta M} \frac{\Delta f_o}{\delta_s} \]  

(7)

where \( \delta_s \) is the skin depth of the metal shields. The advantage of this equation is that it does not need to deal with the complex field expressions as does (5a). The disadvantage is the requirement of very accurate prediction of the frequency shift because of the change in distance. Fig. 3 is the comparison of (5a) and (7).

Figure 2 Relationship of resonant frequency and gap dimension with \( D/L = 10 \text{ mm}/5 \text{ mm} \) and \( M = L_1 = L_2 \)

Figure 3 Comparison of calculated \( Q_c \) s by using equations (5a) and (7)
and (7) with a larger distance between the sample and shield shows (7) is not adequate for larger \( M \) values.

For the calculation of power dissipated by the side regions \( (P_r) \), the maximally possible power is computed by the integration of the Poynting vector over the surface of \( r = b \) (\( b \) is the radius of metal shields) by assuming a perfect absorber for this surface [28].

The relation of \( Q_c \) and \( Q_r \) with the sample dimension, dielectric constant and distance between the shields are shown in Figs. 4 and 5, respectively. In Fig. 5, a shield diameter of 10 cm was used. It can be easily understood that in Fig. 4 the conductor loss increases with increasing the \( D/L \) ratio because of the increasing surface area of the sample’s cross section. In addition, at a certain small value of \( M \) one can see intersections of curves for different values of \( L \) in Fig. 4. With the same \( D/L \) value and large \( M \) values, the higher conductor loss (lower \( Q_c \)) for larger \( L \) is because of the larger metal loss area in regions two and three where the surface current is much larger than that in areas four and five. As \( M \) approaches zero, the higher \( Q_c \) for a larger \( L \) value is because the effect of a larger sample volume for energy store is more obvious than that for a smaller \( L \) value. In Fig. 5, the lower radiation loss for a higher dielectric constant is because of the lower transmission coefficient on the dielectric/air boundary and smaller field amplitude outside the sample. Three kinds of intersections of curves are found in Fig. 5. First, the \( D/L \) value is the same. With small \( M \) values, the higher radiation loss (lower \( Q_r \)) for larger \( L \) is because of the larger side region for field radiation. As the \( M \) value increases, the higher \( Q_c \) for larger \( L \) is because the smaller \( M/L \) ratio in the side region reduces the radiation loss. Secondly, the \( L \) value is the same. For small \( M \) values, the lower \( Q_r \) for larger \( D \) value is because the shorter \( b - a \) in the regions four to six causes larger electromagnetic fields at \( r = b \) and higher radiation loss. For large \( M \) values, the higher radiation loss for a smaller \( D \) value is because the volume for energy store is smaller. Finally, the \( D \) value is the same. If the \( M \) value is small, the radiation loss is higher for larger \( L \) because of the larger side region for field radiation. On the other hand, the radiation loss is lower for larger \( L \) with large \( M \) values. The reason is the smaller \( M/L \) ratio in the side region for radiation loss.

4 Experiments

To confirm the validity of loss calculations of the modified model in previous discussions, experiments are conducted to examine the results of computations. The computation results of the unmodified model also will be obtained to compare with those of the modified one. The experimental setup is shown in

![Figure 4: Relationship of \( Q_c \) with various sample and air gap dimensions for \( \varepsilon' = 20 \) and shield conductivity \( 1.41 \times 10^5 \text{ S/m} \)]
Three ceramic specimens with dielectric constants 23, 10 and 37 and the quality factors because of dielectric loss $Q_d = 8.2 \times 10^7/f (\text{GHz})$, $1.24 \times 10^5/f (\text{GHz})$, and $6.4 \times 10^3/f (\text{GHz})$, respectively, are adopted for loss measurements. The above $Q_d$ s are measured by the cylindrical dielectric resonator technique as shown in Fig. 7 [1] with the assumption of $f \times Q_d = \text{constant}$ [29]. The diameters and thicknesses are 11.48/3.33, 12.66/4.36 and 10.72 mm/3.06 mm, respectively. Two brass plates with conductivity $1.41 \times 10^7 \text{S/m}$ and diameter 10 cm are used for metal shields. The conductivity was measured by the methods suggested by [28, 30].

The quality factor of the system $Q_T$ is calculated by

$$Q_T = \frac{f_0}{\Delta f_{3dB}} \frac{1}{|1 - S_{21}|} \quad (8)$$

where, $\Delta f_{3dB}$ and $S_{21}$ are the measured 3 dB bandwidth and transmission coefficient, respectively, as shown in Fig. 8. Fig. 9 gives the computed $Q_c$ and $Q_r$ of the ceramic with dielectric constant 23 by using (5) for $h < \lambda/2$. Obvious differences can be observed between the calculation results for the modified and unmodified models. Similar situations happen to the specimens with dielectric constants 10 and 37. By using the results in Figs. 9, 10 gives the total quality factors $Q_T$ for the measured results and the calculated results of the modified and unmodified
models. From the experimental results, at first the $Q_T$ values increase with increasing the distance $M$ between the shields and sample. This is expected because of the decreasing conductor loss. After the $Q_T$ values reach a peak, they decrease with increasing the $M$ value because the radiation loss starts dominating the loss mechanism. It can also be found that the computational results by using the unmodified Itoh and Rudokas model cannot correctly describe the radiation loss because the decay on the modified Bessel function is too fast. By using the modified (3), the decay of the modified Bessel function is much closer to the practical situation. Although the modification method is still not perfect, as the experimental results do not exactly fall on the theoretically calculated maximally possible solid curve in Fig. 10, it does give a dramatic improvement to the unmodified model, even though the boundary conditions are not fully satisfied for this simplified model as mentioned before.

With the reasonable prediction of conductor and radiation losses, the dielectric loss of the dielectric resonator sample can be measured by this parallel-plate dielectric resonator setup by using (4) [2]. From Fig. 4, if $D/L < 2$ and $h/L > 3$ ($M > L$), the conductor loss is less than one-tenth of the loss tangent with the dielectric loss factor of a sample larger than $10^{-4}$. The low radiation loss can also be kept under the condition of $\epsilon' > 10$ and $M < 4$ mm, as can be seen in Fig. 5. For $\epsilon' < 10$, in order to keep radiation loss low, high conductor loss will occur by reducing the distances between the shields and sample. In practical measurement, the choice of

Figure 9 Calculated $Q_e$ and $Q_r$ by the modified and unmodified Itoh and Rudokas model

Figure 10 Computed and experimental results of quality factors
measurement setup should ensure lower summation of radiation and conductor losses to achieve higher measurement accuracy. From (2) and (3), the measurement of the dielectric constant can be conducted.

5 Summary

The various aspects of loss for the TE\textsubscript{01\textalpha} mode of the parallel-plate dielectric resonator system have been studied. The modified Itoh and Rudokas model of the parallel-plate dielectric resonator was adopted for study. This modified model has facilitated a complete description of the relationship of conductor and radiation losses with various setup dimensions. For the calculation of the conductor loss factor, the perturbation method has shown lower accuracy than the traditional method of (5a) for larger sample/shield distances. Summations of conductor and radiation losses with the added dielectric loss are compared with the practically measured total loss factor. A comparison between the unmodified and modified models is done; this modified model has shown a dramatic improvement over the unmodified one. Finally adequate setup dimensions should be considered to ensure the accepted summation of conductor and radiation losses for dielectric loss measurement.

6 Acknowledgment

This work was supported by the National Science Council of the Republic of China, Taiwan under Contract No. NSC 91-2213-E-161-004.

7 References


