Measurements of microwave dielectric properties by an amended cavity perturbation technique

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A R T I C L E   I N F O
Article history:
Received 7 October 2007
Received in revised form 27 March 2008
Accepted 28 March 2008
Available online 7 April 2008

Keywords:
Cavity perturbation method
Complex permittivity
Dielectric constant
Dielectric loss
Loss tangent
Quality factor
Electromagnetic coupling

A B S T R A C T
The accuracy of an amendment theory on the cavity perturbation technique for microwave dielectric properties measurements has been studied. The accuracy was confirmed by comparisons of experimental results with those measured by another well known measurement method – post resonance technique. Based on the conventional circular aperture coupling structure, a new equation is proposed to determine the quality factor of TE modes for the rectangular cavity resonator. The relation of quality factor with the aperture radius was studied for determining the adequate aperture diameter.

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1. Introduction

Cavity perturbation technique was widely adopted for microwave dielectric properties measurements [1–8]. Both the formulas for the calculations of dielectric constant and loss tangent, and the procedure of measurement are very simple [9–13]. The fundamental concept of perturbation technique is that the presence of a small piece of dielectric sample in the resonant cavity will cause a shift of resonant frequency and a decrease of the quality factor of the cavity. The decrease in the quality factor of the cavity is because of the presence of sample’s dielectric loss. The dielectric constant and loss tangent of the specimen can then be calculated from the changes of frequency and quality factor [9–13]. The advantages and disadvantages of the cavity perturbation techniques, as well as the comparison with other resonance techniques, have been given in Ref. [3]. However, the assumption of decreasing quality should be modified. In fact, the quality factor of the cavity will increase if the specimen is lossless [6]. Ref. [3] did not deal with the condition of increased quality factor. This phenomenon was then introduced in Ref. [14] to amend the measurement uncertainty of the conventional perturbation theory. Direct calculation formulas to simplify the procedure of permittivity measurements and amend the uncertainty on loss tangent measurement by the conventional formulas were also suggested in this reference. On the other hand, there are still some important issues not included in Refs. [3] and [14] and should be investigated. First, the amendment formula of Ref. [14] is lack of experimental confirmation. Second, the relationship of the cavity and sample dimensions with the improvement of uncertainty by the amended formulas will to be helpful to understand the difference between the conventional and amended formulas. Finally, the study of electromagnetic fields coupled into the conducting cavity through an aperture is important in the determination of cavity’s quality factor. Various approaches to determine the field coupled through a small aperture in a conducting wall.
between two regions have been published. The generalized formulations for aperture coupling problem have been given [15–19]. The electromagnetic coupling of an incident wave to a cavity has been investigated [20–24]. The quality factors of various cavity geometries through a slot or circular aperture coupling were also studied [25–32]. A mathematical equation relating the quality factor and the cavity, and aperture dimensions for a two-port coupling is still lacking in the above literatures and will be given in this paper.

The widely used circular aperture coupling method and rectangular cavity are investigated in this paper. To examine the accuracy of the amendment on conventional perturbation theory, samples are measured by this cavity perturbation technique under a rectangular cavity at 10.16 GHz. Experiments will also be conducted by another measurement method to confirm the measured results. The relationship of the measurement uncertainty without the amendment and the volume ratio of cavity to sample for the rectangular cavity will be studied. Furthermore, theoretical analysis of circular aperture coupling to rectangular cavities will be investigated at both X-band and K-band frequencies. A mathematical equation relating the quality factor and the cavity dimensions and aperture radius will be derived. The choice of aperture dimension is then discussed.

2. Theoretical analysis

2.1. Dielectric properties measurements

For the rectangular cavity in Fig. 1, the TE_{10N} modes are used for the complex permittivity measurements [1,2]. The sample is placed in the positions of maximum intensity of electric field. \( N = \text{odd} \) is always adopted because sample position can be located easily as the geometry center of the cavity is one of the maximum positions. The modified formulas for dielectric constant and loss tangent measurements were proposed [14]

\[
\varepsilon' = \frac{V_c(f_c - f_i)}{2V_s f_i} + 1 \tag{1}
\]

\[
\varepsilon'' = \frac{V_c}{4V_s} \left( \frac{1}{Q} - \frac{1}{Q_c} \right) \tag{2}
\]

\[
Q_c' = Q_c \left[ 1 + \left( \varepsilon' - 1 \right) \frac{V_s}{V_c} \right] \tag{3}
\]

\[
1 = \frac{Q_d}{Q_c} = \frac{\varepsilon''}{\varepsilon'} \tag{4}
\]

where \( f_c \) and \( f_i \) are the resonant frequencies, and \( Q_c \) and \( Q_s \) are the measured unloaded quality factors of the cavity without and with a sample inside the cavity, respectively. \( V_c \) and \( V_s \) are the volumes of cavity and the sample. The \( \varepsilon_r = \varepsilon - j\varepsilon' \) is the relative complex permittivity. Before inserting the sample in the cavity, the cavity’s quality factor is \( Q_c \). \( Q_c' \) is the theoretical quality factor if the inserted sample is “lossless”. In conventional cavity perturbation theory, it is assumed that the presence of a specimen in the metal cavity would decrease the quality factor of the cavity because of the presence of sample’s dielectric loss. This concept is used to calculate the loss tangent (\( \tan \delta = 1/Q_d \)) of the sample. In fact, the quality factor of the cavity will increase if the specimen is lossless (\( Q_c' \)) because the quality factor is proportional to the dielectric constant \( \varepsilon \). This phenomenon was not considered by the conventional theory where \( Q_c \) is adopted instead of \( Q_c' \) in Eq. (2), i.e., \( Q_c' \) is equal to \( Q_c \) in Eq. (3). Fig. 2 shows the measurement uncertainty on \( Q_d \) under the conditions of \( Q_c = 2000 \) and \( Q_d = 3000 \). The measurement uncertainty becomes more obvious as the sample volume \( V_s \) is increased where the difference between \( Q_c' \) and \( Q_c \) is larger.

2.2. Cavity coupling by circular apertures

The measured loaded quality factor of a cavity can be expressed as

\[
1 = \frac{1}{Q_L} - \frac{1}{Q_c} - \frac{1}{Q_e} \tag{5}
\]

\[
Q_L = \frac{f_0}{\Delta f} \tag{6}
\]

Fig. 1. TE cavity perturbation technique. The \( a, b, \) and \( d \) are the width, height, and length of the cavity, respectively.
where $Q_c$ and $Q_e$ are quality factors due to conductor loss (unloaded quality factor) and power coupled to external circuit; $f_o$ and $D_f$ are the resonant frequency and one half power bandwidth, respectively.

For the two-port coupled rectangular cavity in Fig. 1, by the similar procedures in [25], we can derive the theoretical $Q_e$ of $TE_{10}$ mode,

$$Q_e = \frac{(kad)^2 b \eta}{2\pi^2 R_s (2N^2a^3b + 2bad^3 + N^2a^3d + ad^3)}.$$  \hspace{1cm} (8)

This equation can be found in a lot of electromagnetic wave literature, where $c$ is the speed of light in free space, $\eta$ is the intrinsic impedance, and $R_s$ is the surface resistance of the cavity. A metal conductivity of $1 \times 10^7$ S/m is adopted for $Q_c$ calculations. For the X-band cavity, a typical $d/(cm)/N = 2$ with $f_o = 10$ GHz is adopted, for example, $TE_{105}$ mode for $d = 10$ cm. For the K-band cavity, a typical $d/(cm)/N = 1$ with $f_o = 20$ GHz is adopted, for example, $TE_{109}$ mode for $d = 9$ cm. It is quite clear that it will be difficult to achieve a high degree of loading unless a larger aperture is used. However, $Q_e/Q_c$ is expected to reduce the effect of $Q_e$ on measuring unloaded quality factor $Q_c$. For example, if we increase aperture diameter $2r$ of the X-band cavity to 6 mm, we can then have $Q_e = 588$ which would be significantly smaller than the unloaded quality factor $Q_c$ of the cavity. From Fig. 3, the adequate values of aperture diameter are 3 and 1.5 mm for X-band and K-band cavities, respectively.

### 3. Experiments

To examine the validity of using this amended perturbation technique, samples were measured by a X-band cavity fabricated by standard WR-90 copper waveguide with $d = 13.5$ cm. The measured frequency is 10.16 GHz for mode $TE_{107}$. To confirm the accuracy of measured results, the dielectric constant and loss tangent of those samples were also measured by the widely used post resonance technique [3,33,34] as shown in Fig. 4.

The measurement results are listed in Table 1. Good agreement in dielectric constant (real part) measurements was found between the post resonance technique and the cavity perturbation technique, which confirms the accuracy of dielectric constant measurements. The table shows $f/Q_d(f\tan\delta)$ values as well as $Q_d$ values to facilitate comparisons between the cavity perturbation and post resonance methods. The use of $f/Q_d$ for comparison is based on the assumption that the loss tangent is proportional to the frequency ($f/Q_d = \text{constant}$) at microwave frequencies [35]. This relationship has been proven accurate in some examples [3], and can effectively compare the accuracy of loss tangent measurement at various measurement frequencies. Good agreement in loss tangent measurements was also reached between two techniques and the accuracy in measured loss tangent values was proved. It can be seen that the $f/Q_d$ values obtained from the mod-

![Fig. 2. The relationship of the measurement uncertainty without the amendment of $Q_c$ and the volume ratio of cavity to sample.](image2)

![Fig. 3. Comparisons of $Q_c$ and $Q_e$ for the X-band and K-band cavities.](image3)

![Fig. 4. The post resonance technique.](image4)
ified formulae are much closer to the measured results by the post resonance technique and the accuracy in measured loss tangent values by the modified formulae is proved.

4. Conclusions

Modified formulae for the microwave dielectric properties measurements by the cavity perturbation technique have been studied. This modification simplifies the procedure of permittivity measurements and corrects the uncertainty on loss tangent measurement by the conventional formula. The relation of measurement uncertainty without amendment with the cavity/sample volume ratio was also investigated. A formula for calculating the quality factor due to the aperture coupling has been proposed. The influence of aperture coupling to the quality factor has been examined for both X-band and K-band cavities. The adequate aperture diameters were suggested. By comparison with the post resonance method to confirm the measured results, the modified formula on cavity perturbation technique have shown accurate measurements of both dielectric constant and loss tangent.

Acknowledgement

This work was partially supported by the National Science Council of the Republic of China, Taiwan under Contract No. NSC 92–2213–E–161–003.

References

[27] G. Faby, K. Schunemann, Q-factor measurements of open resonators in the millimeter-wave range including coupling losses, IEEE


