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Study of microwave dielectric properties measurements by various resonance techniques

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Abstract

A systematic study on various resonance measurement techniques of dielectric constant and dielectric loss at microwave frequencies has been undertaken. Characteristics of various resonance techniques are compared with each other. Suggestion on how to select adequate measurement techniques of microwave dielectric properties is given. Practical measurements by different methods are made and the results are compared. The trend for future development is discussed.

Keywords: Microwave measurement; Dielectric constant; Dielectric loss; Loss tangent; Complex permittivity

1. Introduction

Resonance techniques are widely used for measurements of microwave dielectric properties (complex permittivity, $\varepsilon = \varepsilon_r + j\varepsilon_e$) [1–11], i.e., dielectric constant and dielectric loss (loss tangent, $\tan\delta$), which can be divided into two groups: The first is that the resonance is basically supported by the dielectric itself. The sample acts as a dielectric resonator. Metal shields with different geometries are always introduced to prevent radiation loss. This type is called dielectric resonance technique. The second type is that the resonance is supported by the metal walls of a metal cavity. The presence of samples in the cavity causes only a "perturbation" on the field distributions in the metal cavity. The second type is called the cavity perturbation technique. Various resonance techniques have been published. In order to make an adequate choice among those techniques, a systematic study and comparison to various methods is necessary but not enough work has been done on this [12]. In this paper, four most widely used and reliable resonance methods with three dielectric resonance techniques and one cavity perturbation technique

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will be studied. Samples will be measured by those four methods to compare the measured results. The work should be done in the future is also introduced.

2. Measurement techniques

2.1. Dielectric resonance techniques

There are various dielectric resonance techniques for measuring the dielectric properties of dielectric samples. The lowest TE mode of cylindrical dielectric sample is always used for measurements because it is easy to identify the resonant peak, and the calculation equations for the dielectric properties are more easily derived than those of other modes. The main advantage of the dielectric resonance methods is the higher accuracy of measurements, but there are several disadvantages. Usually, only a single frequency point can be measured for each sample. The dielectric resonance techniques do not have the swept frequency capability. And, the calculations of dielectric properties are usually very complicated; a computer program is always required to deal with the complicated Bessel functions. The third disadvantage is the requirement of sample dimension; much larger sample volume is needed than that for the perturbation technique. However, the dielectric resonance technique are still widely used because of high accuracy of dielectric properties measurements, especially for dielectric loss measurement, in comparison to other methods. Three most widely used dielectric resonance techniques are discussed in the followings: (i) Post resonance technique, (ii) Cylinder cavity resonance technique, and (iii) Waveguide reflection resonance technique. The difference between those techniques are based on different geometrical arrangements of metal shields.

2.1.1. Post resonance technique

Originally suggested by Hakki and Coleman in 1960 [7], this method has been widely used and has become the most popular dielectric resonance method for measuring the dielectric properties of dielectric samples. A cylindrical dielectric rod is placed between two parallel metal plates as shown in Fig. 1. Two coupling antennas are used to couple the power in and out. The solid arrows in the figure indicate the direction of signal flow. The TE_{011} mode is adopted for measurements. The resonant frequency f_o, the half power bandwidth Δf_{3dB}, the insertion loss S_{21}, and the diameter $D(=2a)$ and thickness L of the specimen are recorded for the calculations of dielectric properties. The dielectric constant is calculated by the following equations of TE_{011} mode [13],

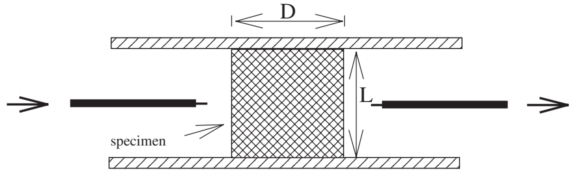

Fig. 1. Post resonance technique.

$$\varepsilon_r = \left(\frac{\lambda_o}{2\pi}\right)^2 (k_{ci}^2 + k_{co}^2) + 1 \tag{1a}$$

$$k_{co}^2 = \left(\frac{2\pi}{\lambda_o}\right)^2 \left[\left(\frac{\lambda_o}{2L}\right)^2 - 1\right] \tag{1b}$$

$$\frac{J_0(k_{ci}a)}{J_1(k_{ci}a)} = -\frac{k_{co}a}{k_{ci}a}\frac{K_0(k_{co}a)}{K_1(k_{co}a)} \tag{1c}$$

$$\lambda_o = \frac{c}{f_o} \tag{1d}$$

where k_{ci} and k_{co} are sample geometry dependents, J and K are Bessel function and modified Bessel function, respectively.

The computation of dielectric loss is given by [13],

$$\tan\delta = A\left(\frac{1}{Q_u} - \frac{1}{Q_c}\right) \tag{2a}$$

$$Q_u = \frac{f_o}{\Delta f_{3dB}}\frac{1}{1 - S_{21}} \tag{2b}$$

$$Q_c = \frac{A}{BR_s} \tag{2c}$$

$$A = \frac{W}{\varepsilon_r} + 1 \tag{2d}$$

$$B = \left(\frac{\lambda_o}{2L}\right)^3 \frac{1+W}{30\pi^2\varepsilon_r} \tag{2e}$$

$$W = \frac{J_1^2(k_{ci}a)}{K_1^2(k_{ci}a)} \frac{K_0^2(k_{ci}a)K_2^2(k_{ci}a) - K_1^2(k_{ci}a)}{J_1^2(k_{ci}a) - J_0^2(k_{ci}a)J_2^2(k_{ci}a)} \tag{2f}$$

where Q_u is the measured unloaded quality factor, Q_c is the quality factor due to conductor loss, R_s is the surface resistance of metal shields. The factor A is the ratio of total energy stored in the dielectric and air to the energy stored in the dielectric. Usually the A factor is very close to unit for dielectric constant larger than 20, which means most of the energy is stored in the dielectric. The radiation loss is neglected in measurement. The accurate calculation of conductor loss is a critical point for correct computation of dielectric loss for this method. The surface resistance R_s of the conducting shields may change with the surface roughness, oxidation, scratch, and temperature variation. Dielectric loss lower than 5×10^{-4} is not recommended for measurement by this post resonance method unless the R_s value can be precisely determined [14]. The extra measurement on the surface resistance is one defect of this technique. The other disadvantage of the resonant post method is the requirement for the sample's dimension is largest among the three dielectric resonance techniques. For instance, for a material with dielectric constant 20, a diameter 10 mm and thickness 5 mm specimen would be required for the measurement at 10 GHz. For a single crystal, this dimension can be quite difficult to prepare. The main advantage for this method is that equations shown above for the calculations of dielectric properties were well developed and they gave very reliable measurement on dielectric constant value. Therefore the technique is still used by a lot of people.

2.1.2. Cylinder cavity resonance technique

The setup of the cylinder cavity dielectric resonance technique is shown in Fig. 2. The technique is mainly used for loss tangent measurements of low loss materials [8,9]. The sample under test is put inside a cylindrical metal cavity with diameter d and height h. The $TE_{01\delta}$ mode is used for measurement. Eqs. (2a) and (2b) are used the calculation of dielectric loss. Since the sample does not contact with the metal shields, the conductor loss is much lower than that of the post resonance technique. The accuracy for dielectric loss is higher. If d/D and h/L are larger than 2, the conductor loss be neglected with loss tangent of specimen larger than 2×10^{-4} [15]. In addition, the sample dimension requirement is smaller than that for the post resonance technique. For a sample with dielectric constant 20, measured at 10 GHz, the $D \times L$ is about 6 mm × 3 mm—much smaller than the 10 mm × 5 mm requirement for the post resonance method. The lack of adequate expressions to measure dielectric constant is one disadvantage. Furthermore, the specimen is put inside a closed cavity, the desired $TE_{01\delta}$ mode may be interfered and confused by the modes supported by the metal cavity.

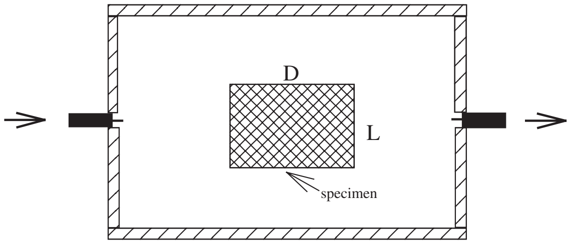

Fig. 2. Cylinder cavity resonance technique.

2.1.3. Waveguide reflection resonance technique

In contrast to the above two techniques where the transmission signal is taken, a reflection cavity dielectric resonance technique is used in which case the reflection signal is measured. The setup is shown in Fig. 3. The sample is put inside a rectangular microwave waveguide with one end shorted and adjustable for the best resolution of resonance signal. The $TE_{01\delta}$ mode is also used for this method. Like the cylinder cavity method, this technique has been used for low loss measurement [10]. Eq. (2a) is also used for the calculation of dielectric loss. The unloaded Q is calculated by using the expression [16],

$$Q_u = \left(\frac{P_x - P_{min}}{P_{max} - P_x}\right)^{1/2} \frac{2}{2 \pm \sqrt{P_{min}} - \sqrt{P_{max}}} \frac{f_o}{\Delta f_x} \tag{3}$$

The ± sign accounts for the undercoupled case (+) and overcoupled case (−) and $P_{max} \approx 1$. Expression for the dielectric constant is also not

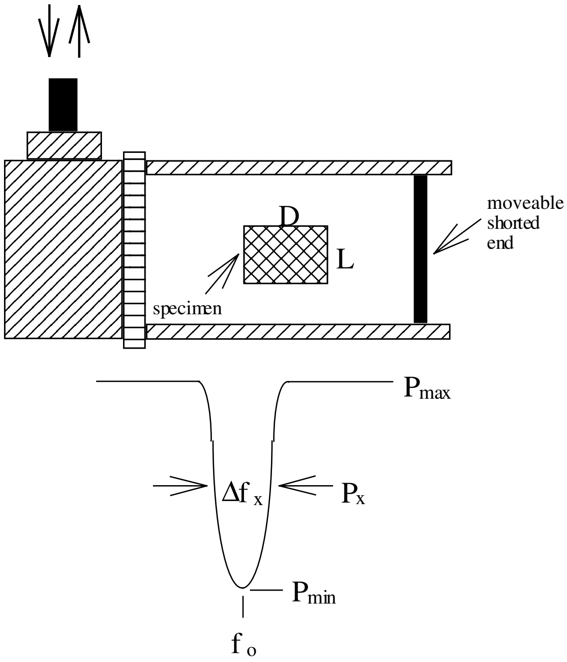

Fig. 3. Waveguide reflection resonance technique.

available for this method. The conductor loss is the same order as that of the cylinder cavity resonance technique. This reflection method also has the advantage of smaller required sample dimension. It has all the disadvantages that the cylinder cavity method has. In addition, measurement error of the reflection method is higher than that of the cylinder cavity technique because of the uncertainty condition that will be discussed later.

2.1.4. Error analysis of dielectric constant measurement

There are three major error sources for the dielectric constant measurements for the post resonance technique. The first one ($E_{\varepsilon 1}$) is the error caused by the measurement of resonant frequency. The $E_{\varepsilon 1}$ can be easily derived from (1a),

$$E_{\varepsilon 1} = 2\frac{\Delta f_o}{f_o} \tag{4}$$

where Δf_o is the measurement error on resonant frequency f_o.

The other error source is the measurement error of the sample dimension. The dielectric constant

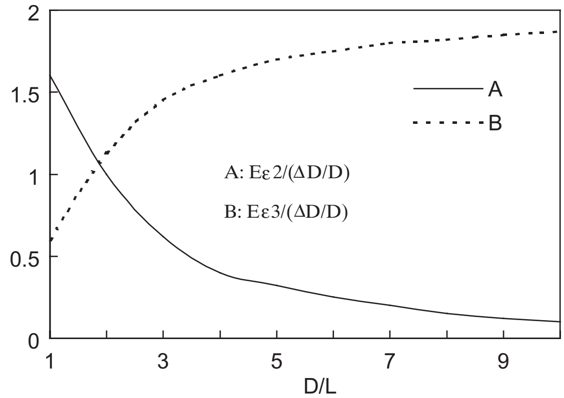

Fig. 4. Normalized measurement errors of dielectric constant of the post resonance technique due to the error on dimension measurements.

measurement error ($E_{\varepsilon 2}$) from the diameter measurement can also be derived from (1a),

$$E_{\varepsilon 2} = \frac{2k_{ci}^2}{\varepsilon_r k_o^2}\frac{\Delta D}{D} \tag{5}$$

The dielectric constant measurement error ($E_{\varepsilon 3}$) from the thickness measurement can be expressed as [17]

$$E_{\varepsilon 3} = (1 + W)\frac{\lambda_o^2}{2\varepsilon_r L^2}\frac{\Delta L}{L} \tag{6}$$

The relationship of $E_{\varepsilon 2}$ and $E_{\varepsilon 3}$ and the sample dimension are given in Fig. 4. With typical sample dimension $D/L = 2 \sim 4$ for measurement, a 1% error on the diameter or thickness measurement can cause a measurement error of about 0.5–1.5%.

The last error source is the uncertainty error. That error may arise from different reasons: calibration error of the network analyzer, temperature and humidity instability, bends on the measuring cables, exact position and imperfection of the specimen, and other unexpected reasons.

2.1.5. Error analysis of loss tangent measurement

There are various sources of error for the loss tangent measurement. The first main error source is from the measuring error on the unloaded quality factor Q_u, by (2b), which can be caused by the measurement errors of half power bandwidth, amplitude, and transmission coefficient. The error

($E_{\delta 1}$) from the frequency bandwidth measurement error $\Delta(\Delta f_{3\text{dB}})$ can be derived from (2) and expressed as

$$E_{\delta 1} = \left(1 + \frac{A}{Q_c \tan\delta}\right)\frac{\Delta(\Delta f_{3\text{dB}})}{\Delta f_{3\text{dB}}} \tag{7}$$

For the cylinder cavity and the reflection resonance techniques, because of the low conductor loss ($Q_c \tan\delta \gg A$), the $E_{\delta 1}$ is mainly controlled by the bandwidth error. However, for the post resonance method, the second term in bracket cannot be neglected. The error from the amplitude resolution error (ΔR in dB) for the post and cylinder cavity resonance techniques is calculated by using the expression [16],

$$E_{\delta 2} = 0.23\left(1 + \frac{A}{Q_c \tan\delta}\right)\Delta R \tag{8}$$

For low conducting loss condition, a 1% amplitude resolution error can cause a 0.23% error on loss tangent measurement. For the reflection method, the amplitude resolution error is [16],

$$E_{\delta 2} = 0.115\frac{1+\gamma}{\gamma}\frac{\gamma P_{\max} + P_{\min}}{P_{\max} - P_{\min}}\Delta R \tag{9a}$$

$$\gamma = \frac{P_x - P_{\min}}{P_{\max} - P_x} \tag{9b}$$

The amplitude resolution of the reflection method depends on how sharp the trough is. $E_{\delta 2}$ for the reflection method vary from 0.5% to 1% for the depth of the trough changing from 5 dB to 40 dB. With a similar procedure as that of $E_{\delta 1}$ and $E_{\delta 2}$, the measurement error because of the transmission factor measuring error ΔS_{21} can be expressed as

$$E_{\delta 3} = \left(1 + \frac{A}{Q_c \tan\delta}\right)\frac{\Delta S_{21}}{1 - S_{21}} \tag{10}$$

which indicates that a strong coupling between the device under test and the testing instrument will increase the error on loss tangent measurement because the S_{21} will approach unit as the coupling increases.

The other main error source is from the calculation errors on both factor A and conducting loss. For the calculation error of factor A, the loss tangent measurement error can be derived and expressed as

$$E_{\delta 4} = \left(1 + \frac{A}{Q_c \tan\delta}\right)\frac{\Delta A}{A} \tag{11}$$

Usually this error can be neglected for high dielectric constant samples. The loss tangent measuring error caused by the conducting loss calculation can be expressed as [17]

$$E_{\delta 5} = \frac{A}{Q_c \tan\delta}\frac{\Delta R_s}{R_s} \tag{12}$$

where ΔR_s is the measurement error of the surface resistance of metal shields. For low conducting loss condition, the $E_{\delta 5}$ can also be neglected.

The last one ($E_{\delta 6}$) is the uncertainty of measurements. As discussed above, that error is caused by different factors. The uncertainty of measuring the power level is important in the reflection method which is calculated from [16],

$$E_{\delta 6} = 0.5\frac{(1+\gamma)^{3/2}}{\gamma}\frac{(\gamma P_{\max} + P_{\min})^{1/2}}{P_{\max} - P_{\min}}\Delta P \tag{13}$$

A 1% power level error can cause the uncertainty error $E_{\delta 6}$ larger than 2%.

Finally, the combined rms errors for both dielectric constant and loss tangent measurements can be calculated by using,

$$E_{\varepsilon,\delta}^2 = \sum_i E_{(\varepsilon,\delta)i}^2 \tag{14}$$

with the assumption that all errors are independent.

2.2. Cavity perturbation technique

The fundamental concept of perturbation technique is that the presence of a small piece of dielectric sample in the resonant cavity will cause a shift of resonant frequency and a decrease of the quality factor of the cavity. A rectangular cavity is usually adopted as shown in Fig. 5. The TE_{01N} (N is integer) modes are widely used for dielectric properties measurements. A small piece of rod, sheet, or bar shaped sample is placed in the position of maximum electric field. Odd modes (N:odd) are more often used because of the geometrical center is always one of the maximum electric field positions.

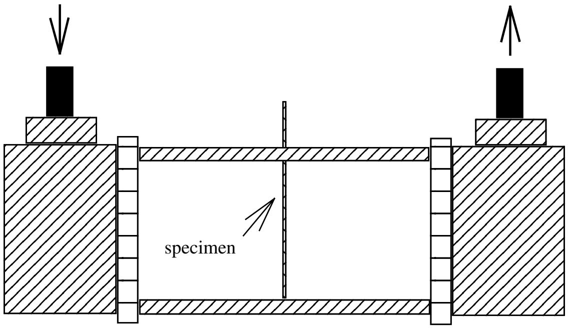

Fig. 5. Cavity perturbation technique.

The dielectric constant and loss tangent of the specimen can then be calculated from the changes of resonant frequency and quality factor of the metal cavity [11],

$$\varepsilon_r = \frac{V_c(f_c - f_s)}{2V_s f_s} + 1 \tag{15}$$

$$\varepsilon_e = \frac{V_c}{4V_s}\left(\frac{1}{Q_s} - \frac{1}{Q_c}\right) \tag{16}$$

$$\tan\delta = \frac{\varepsilon_e}{\varepsilon_r} \tag{17}$$

where f_c and f_s are the resonant frequencies, and Q_c and Q_s are the quality factors of the cavity without and with sample inside the cavity, respectively; V_c and V_s are the volumes of cavity and sample, respectively.

For dielectric constant measurement, the error is mainly from the measurement errors on frequency shift and volume ratio. When carefully handled, the dielectric constant measurement error can be controlled inside 2%. The precision of loss tangent measurement is very difficult to control, which is dominated by the frequency shift error and especially the quality factor change error. This quality factor change error deteriorates the measurement accuracy of dielectric loss because of the combination of various factors—homogeneity of the specimen, coupling condition, specimen holes, and uncertainty. As much as 50% loss tangent measurement error can exist for a sample with $\tan\delta \sim 4\times10^{-4}$ [11]. Loss tangent less than 1×10^{-3} is not recommended to be measured by this technique. Although the cavity perturbation method has the limitation of low accuracy on low loss measurement, the technique has several additional advantages. Unlike the dielectric resonance techniques, there is really no severe tolerance limit on the shape and dimension of measured specimen. The required specimen size is much smaller than those of the dielectric resonance methods. The thickness of the sample is only around 1 mm. The height of the waveguide is 10.2 mm for X-band (8.2–12.4 GHz) measurements. The sample is much easier to be prepared. In addition, the calculations of dielectric properties are very simple while the dielectric resonance techniques need computer program for computations. Furthermore, the cavity perturbation method can measure several frequency points with various TE_{10N} modes, but only single frequency point can be measured for the dielectric resonance techniques.

3. Experiment

To compare the accuracy and adequacy of various measurement techniques, the dielectric constant and loss tangent of two samples were measured by the four most widely used resonance techniques discussed in this paper. Two brass plates with diameter 10 cm and conductivity 1.41×10^7 S/m [14], a cylindrical copper cavity with inside diameter 3 cm and height 1.7 cm, a rectangular copper waveguide section with cross section 4.75 cm × 2.21 cm, and a rectangular copper cavity with dimension 13.5 cm × 2.29 cm × 1.02 cm were adopted for measurements by the post resonance, cylinder cavity resonance, waveguide reflection resonance, and cavity perturbation techniques, respectively. The measurement results are listed in Table 1. Good agreement in dielectric constant measurements was found between the post resonance technique and the cavity perturbation technique, which confirms the accuracy of dielectric constant measurements by those two methods. For the measurements of loss tangent, since the loss tangent increases with increasing the frequency [18], higher loss tangent values for the post resonance method is reasonable because of the higher measured frequencies. Good agree-

Table 1
Dielectric properties measured by various resonance techniques

	Technique	Frequency (GHz)	ε_r	$\tan\delta$
Sample#1	Post resonance	10.9	23.3	1.65×10^{-4}
$D = 11.5$ mm	Cylind. cav. resonance	6.55	–	8.13×10^{-5}
$L = 3.33$ mm	Waveguide refl. reson.	6.39	–	7.94×10^{-5}
	Cavity perturbation	8.59	23.1	–
Sample#2	Post resonance	8.56	18.3	1.04×10^{-3}
$D = 12.6$ mm	Cylind. cav. resonance	6.08	–	9.26×10^{-4}
$L = 5.37$ mm	Waveguide refl. reson.	5.89	–	8.55×10^{-4}
	Cavity perturbation	8.59	18.5	–

ment in loss tangent measurements was also reached between the cylinder cavity and the waveguide reflection techniques and the measurement accuracy of methods in measuring loss tangent values was proved.

The variation of measured dielectric constant values for repeating measurements could be controlled inside 1% for both post resonance and cavity perturbation techniques. It can be easily understood that this 1% error is from the frequency measurement error ($E_{\varepsilon 1}$) and uncertainty condition. The errors for the repeated measurements on the loss tangent are from the measuring error of unloaded quality factor Q_u and uncertainty error. For the two transmission methods and the reflection method, the observed values are 2% and 6%, respectively. The higher error for the reflection method is because of the higher uncertainty error.

4. Summary

Three dielectric resonance techniques and one cavity perturbation technique have been discussed and compared. The cylinder cavity resonance method is the best choice for loss tangent measurement but dielectric constant measurement is not available. The cavity perturbation technique is good for dielectric constant measurement and the required specimen is very small but is not adequate for loss tangent measurement. The post resonance technique can measure both dielectric constant and loss tangent but is not adequate for low loss tangent measurement. The choice of those three measurement techniques will depend on the available sample dimension and the accuracy requirement. The waveguide reflection method is not recommended because it has higher measurement error on loss tangent measurement and dielectric constant measurement is not available. From the above discussions, for the cylinder cavity resonance technique, if the theory for measurement of dielectric constant can be accurately developed, it will become the most attractive method for dielectric properties measurements. However, considering the limitation on sample size, cavity perturbation technique will be the best choice.

Acknowledgements

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